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Photon correlation of images through turbulence

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ABSTRACT

We discuss the importance of the *temporal* fluctuation of images viewed through atmospheric turbulence in the determination of their *spatial* correlation functions. If the basic data is considered to be divided into time-frames, then the temporal characteristics simply determine the exposure or integration time per frame. However, it is more fundamental and better from the point of view of signal-to-noise ratio to treat the data as a set of photon coordinates in space and time. For either approach, the temporal correlation function of the image intensity is of great importance and new measurements of this quantity and its wavelength dependence recorded at La Palma, Canary Islands are given.

1. INTRODUCTION

A variety of interferometric imaging techniques are currently being developed and applied in optical astronomy and other situations involving imaging through turbulence[1]. Although such methods have been of interest since the classic studies of Fizeau and Michelson, and more recently of Hanbury Brown and Twiss, the modern renaissance is due to Labeyrie who invented the technique of speckle interferometry[2]. As originally proposed and implemented, this involves the processing of many short-exposure images taken through a fairly narrowband filter (typically on the order of 10-20nm bandwidth) to yield a diffraction-limited estimate of the spatial auto-correlation of the object or, equivalently, its energy spectrum. Extensions of the method to diffraction-limited *imaging* have been proposed and implemented — the Knox-Thompson[3] and triple correlation[4] methods appear to be particularly promising. However, in these extensions, as in the original speckle method and in all other interferometric imaging methods (such as pupil plane interferometry), it is invariably assumed that the starting point is a set of "frames" of data with the exposure or integration time related in some way to the temporal correlation time of the measured intensity. This is a natural approach when the detector is based on television technology with its concept of TV frames, but new photon event coordinate measuring detectors such as the resistive anode device[5] or PAPA[6] are not limited in this way and are capable of providing the spatial (x,y) and temporal (t) coordinates of photon events at sufficiently high data rates (up to 10^5 per second) to be of practical value in astronomy. The question then arises as to how this data should be processed in an optimal way (i.e. with least bias and highest signal-to-noise ratio). The importance of dealing correctly with the temporal properties was first emphasised in Refs 7-9.

2. DATA RECORDED IN FRAMES

When the short-exposure data is collected in frames, as with detectors based on or related to TV technology, the only temporal parameter to be optimised is the exposure time Δt. The effect of exposure time on the speckle interferometry transfer function (i.e.high light level behaviour) was investigated by Roddier and Roddier[10] and Roddier et al[11]. At high light levels, one clearly requires an exposure time Δt very much less than the correlation time τ of the image intensity and in principle there is no lower bound to Δt at sufficiently high light levels — smaller values of Δt give less bias to the measured energy spectrum with no decrease in signal-to-noise ratio. Some experimental measurements of the effect of exposure time were given in Ref 12.

At low light levels, for images dominated by photon noise, the signal-to-noise ratio at a point in the energy spectrum is proportional to the average number of detected photons per frame, to the square root of the number of statistically independent frames used to find the estimate and to the effective speckle transfer function(i.e.that integrated over the exposure time) ; thus for a given total observation time, there is an optimum exposure time which represents a compromise between a large number of detected photons per frame (large Δt) and little bias of the time-integrated transfer function (small Δt). This was first investigated by Walker[13] and O'Donnell and Dainty[8]. In both of these studies, it was assumed that the temporal cross-energy spectrum of the image intensity from a point source was separable into a product of the energy spectrum and the temporal auto-correlation function of the intensity at a point in the image, i.e.

$$\langle I(u,v,t)I^*(u,v,t+t')\rangle = \langle I(u,v,t)I^*(u,v,t)\rangle\; C(t') \qquad (1)$$

where $C(t') = \langle i(x,y,t)i(x,y,t+t')\rangle$, $i(x,y,t)$ is the instantaneous image intensity and $I(u,v,t)$ is its Fourier transform. This assumption is not strictly valid: its degree of validity depends upon whether the temporal fluctuation is dominated by the motion of a single phase screen across the telescope pupil (the Taylor hypothesis) or whether a combination of phase screens moving

at different velocities provides the time variation[8,11,14]. Notwithstanding the fact that the dominant motion of a single layer *can* be observed on occasion[15], it is reasonable to assume that usually the two functions are separable and the results below also support this multiple phase screen theory. If Eq(1) does hold, then it may be shown that the optimum exposure time for the photon limited case is on the order of *twice* the 1/e correlation time of the image intensity, the exact factor depending upon the detailed form of C(t').

3. PHOTON COORDINATE DATA

In this case the data set becomes a list of the x,y and t coordinates of all photons recorded during an observation. Although the storage requirements may still be large (the data rate will be on the order of 15Mby per minute for 60,000 photons per second and 4by per photon), they are not as large as that implied by frame-based recording, at least for small or moderate numbers of detected photons. Starting with this data, one could of course convert it into frames, and thus process the data with several effective exposure or integration times, perhaps using longer integration times for estimating the lower spatial frequencies. However, as discussed in Ref 16, a higher signal to noise is achieved if a *moving* exposure window is used. For example, in Fig 1, the use of a moving time window of length Δt allows the correlation between photons C,D and E to contribute to the signal, whereas the discrete frame approach would ignore this. The increase in signal-to-noise ratio obtained by using a moving time window is on the order of $\sqrt{2}$ for second order correlation and $\sqrt[3]{3}$ for the triple correlation[16]. It is not clear whether the boxcar time window of Fig 1 is optimum — an exponential window has been suggested in another context[9]— but it is clear that the effective window length Δt is closely related to the correlation time τ of the image intensity.

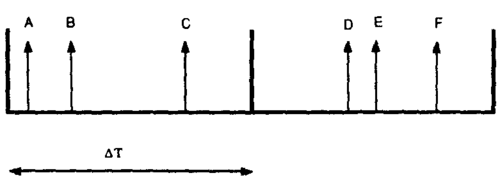

Figure 1

TEMPORAL CORRELATION — THEORY

The nature of the time-evolution depends critically on whether the seeing is dominated by a single turbulent layer that can be considered to move without change across the telescope pupil (the Taylor hypothesis) or whether several layers moving at different velocities are involved[11,17]. If a single layer moves with velocity v over a pupil of diameter D, then the temporal correlation of the image intensity for a point source is equal to the auto-correlation of the pupil function with suitably scaled coordinates, provided that $D \gg r_0$, where r_0 is the Fried seeing parameter. The 1/e correlation time of the image intensity is given by

$$\tau \approx 0.5\frac{D}{v} \quad . \tag{2}$$

For a D = 1 m telescope and v = 10 ms^{-1}, this implies correlation times on the order of 50 ms which is much longer than the range 2-20ms reported in Refs 18-20 and below. Note also that this correlation time is not a function of the wavelength of the light, in contrast to the experimental observations of Scaddan and Walker[19] and those below.

If there are several layers moving at different velocities contributing to the overall fluctuation, then the correlation time is given by[11]

$$\tau \approx 0.36\frac{r_0}{\Delta v} \quad , \tag{3}$$

where r_0 is the Fried parameter and Δv is the standard deviation of the wind velocities of the layers. The parameter r_0 has a wavelength dependence given by[17]

$$r_0 \propto \lambda^{6/5} , \tag{4}$$

so that we may also expect $\tau \propto \lambda^{6/5}$.

TEMPORAL CORRELATION — NEW RESULTS

Measurements of the temporal correlation function of the image intensity for a point source object were made during the nights of May 20-26 1988 inclusive using the 1m telescope at Observatorio del Roque de los Muchachos, La Palma, Canary Islands. The f/15 secondary image was magnified using a x20 microscope objective to give an image scale of 0.69 arc sec per mm: the diffraction-limit at $\lambda = 550$nm is approximately 0.14 arc sec or 200μm at this magnified image plane. All the results reported below were taken with a 65μm aperture at wavelengths/bandwidths of 401/15nm ("blue"), 550/10nm ("green") and 650/23nm ("red"). A photon counting photomultiplier and Langley Ford DC128 correlator were used to sample and process the data, typical sample and experiment times being 0.5ms and 20s respectively. The average detected photon rate was ≈1 per sample time, or 2000 per second, with typical dark counts of <10 per second; no correction for dark count or detector deadtime was necessary.

Figure 2 shows "typical" measured correlations for the intensity fluctuation for nights 1-7 (excluding night 2 when no measurements were made). These curves have been corrected for image wander effects using the procedure described by Scaddan and Walker[19]. The 1/e correlation times τ range from 3.2-8.9ms with an average of 5.9ms. Results taken at another excellent astronomical observing site, Mauna Kea Hawaii, in 1982 gave an average of 15ms over 12 nights for the correlation time, again with a large spread of values[18], whilst measurements by Parry et al[20] at the poor observing site of Herstmonceux (UK) gave values in the range 2-7ms with an average ≈4ms over 10 nights. It is clear from the body of data now accumulating that the speckle correlation time can be such to require effective exposure or frame times ≤ 10ms on many occasions. The seeing parameter r_0 was not measured during our observations, although qualitative estimates of r_0 show no apparent correlation with τ.

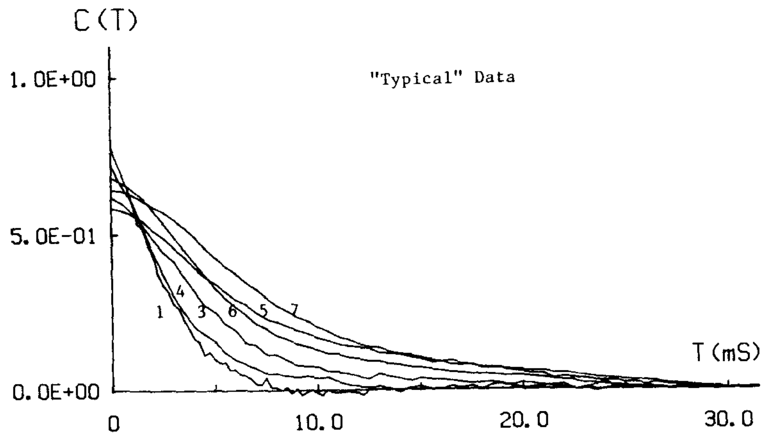

Figure 2

Figure 3 shows the correlation functions for the blue, green and red wavebands defined above, on two different nights. The single layer model predicts no wavelength variation of the correlation time, but the multiple layer theory predicts a $\lambda^{6/5}$ dependence which would mean that the 1/e correlation times were in the ratio 0.68:1.00:1.23. The observed ratios were

0.62:1.00:1.38 and 0.71:1.00:1.20

for nights 4 and 7 respectively, showing reasonable consistency with the multiple layer theory.

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REFERENCES

1 See e.g. *Interferometric Imaging in Astronomy*, ESO-NOAO Workshop Proceedings, Oracle, Az 1987
2 A Labeyrie, Astron Astrophys, **6**, 85 (1970)

3 K T Knox and B J Thompson, Astrophys J, **193**, L45 (1974)
4 A W Lohmann, G Weigelt and B Wirnitzer, Appl Optics, **22**, 4028 (1983)
5 D Rees et al, J Phys E: Sci Instr, **14**, 229 (1981)
6 C Papaliolios and L Mertz, Proc SPIE, **331**, 360 (1982)
7 L Mertz, Appl Optics, **18**, 611 (1979)
8 K A O'Donnell and J C Dainty, J Opt Soc Am, **70**, 1354 (1980)
9 L Mertz, Appl Optics, **23**, 1638 (1984)
10 C Roddier and F Roddier, J Opt Soc Am, **65**, 664 (1975)
11 F Roddier, J M Gilli and G Lund, J Optics(Paris), **13**, 263 (1982)
12 D P Karo and A M Schneiderman, J Opt Soc Am, **68**, 480 (1978)
13 J G Walker, IAU Colloquium #50, *High Angular Resolution Stellar Interferometry*, Maryland, 1978
14 J C Dainty, in *Laser Speckle and Related Phenomena*, edited by J C Dainty, Springer-Verlag, 1984
15 J C Dainty, D R Hennings and K A O'Donnell, J Opt Soc Am, **71**, 490 (1981)
16 M J Northcott, G R Ayers and J C Dainty, JOSA A, **5**, 986 (1988)
17 F Roddier, in *Progress in Optics*, Volume 19, p281, Ed by E Wolf, North Holland,1981
18 K A O'Donnell, B J Brames and J C Dainty, Opt Commun, **41**, 79 (1982)
19 R J Scaddan and J G Walker, Appl Optics, **17**, 3779 (1978)
20 G Parry, J G Walker and R J Scaddan, Optica Acta, **26**, 563 (1979)

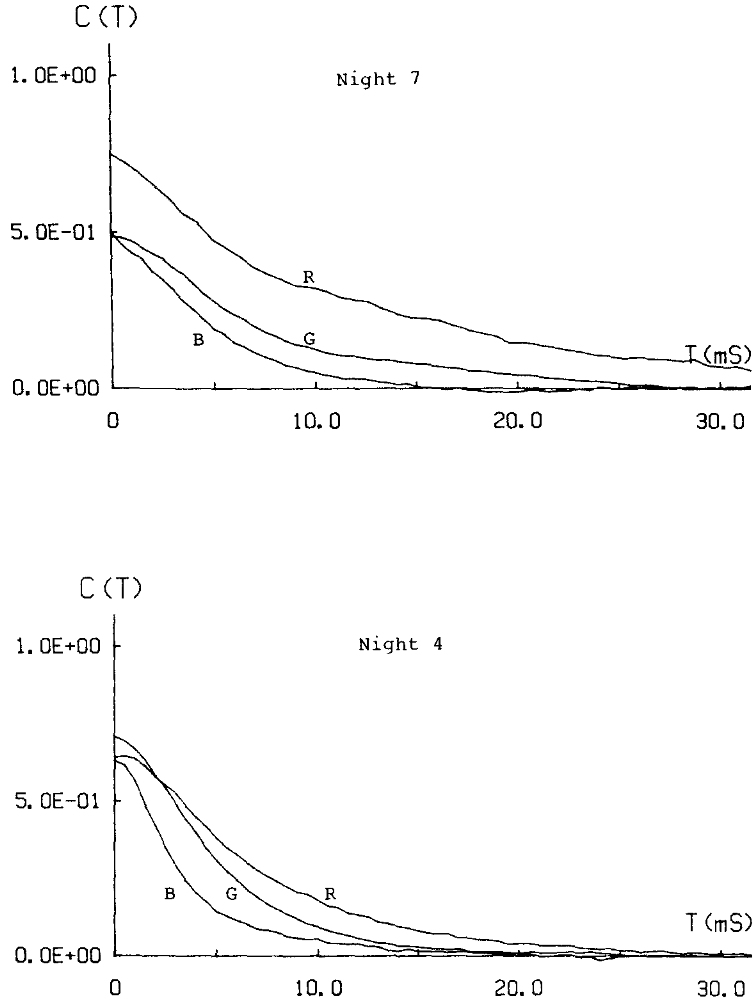

Figure 3