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STATISTICAL PROPERTIES OF ENHANCED BACKSCATTERING PRODUCED BY DENSE COLLECTIONS OF LATEX SPHERES

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ABSTRACT

Measurements of the temporal correlation of both co- and cross-polarised returns around the enhanced backscattering cone of a dense collection of latex spheres are presented. The correlation time shows completely different variation with angle in the two cases and an explanation of this is given. The photon statistics have also been measured for co- and cross-polarised intensities. The normalised factorial moments of the photon counts (which equal to normalised intensity moments) are independent of angle (for angle within $\approx 15mrad$ of backscatter) and detecting polarisations, and they are in agreement with those derived from gamma intensity distribution with the descriptive parameter u=1.20.

1. INTRODUCTION

There has been considerable interest recently in the phenomenon of enhanced backscattering exhibited by dense collections of particulate scatterers such as latex spheres[1-3] and randomly rough surfaces[4]. This phenomenon is of both basic physical importance and potential practical application. Because of the localisation of the photons, transport theory should be modified due to the modification of diffusion constant which is directly related to the localisation effect in certain situations. The coherent backscattering can also find application in analyzing certain characteristics of the scattering media, in remote sensing, etc.

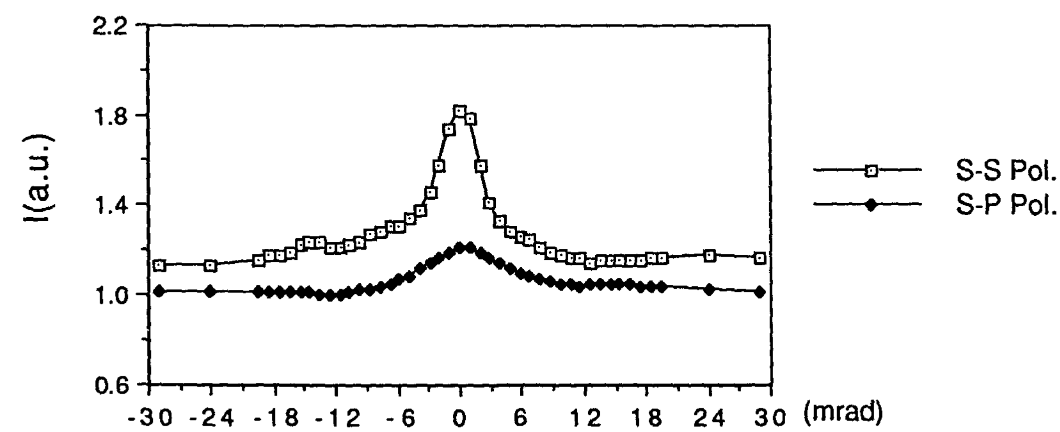

Fig.1. Measured scattered intensity versus angle at and around backscatter showing enhanced backscattering. (S-S Pol.—co-polarised detection, S-P Pol.—cross-polarised detection)

A measurement of coherent backscattering is plotted in Fig.1. As can be seen, in the backscatter direction, for illumination by linearly polarised light , the intensity of the linearly co-polarised scattered component is up to a factor of ~ 1.8 greater than that away from backscatter (i.e. $\approx 30mrad$ away) and the cross-polarised component also shows some enhancement, typically ≈ 1.2 for dense media. These are due to multiple scattering and associated with weak localisation of photons. At and near backscatter, multiple scattering gives rise to a constructive interference of the light travelling along the time reversed paths. For the co-polarised case, the two components of the time reversed paths are identical and therefore give higher enhancement. In the cross-polarised case, the two non-identical components still have correlated phases which give a smaller enhancement.

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Much effort has been put into the study of this phenomenon. The statistical properties of such media, however, are very important in understanding the phenomenon, scattering process and the media itself. Maret and Wolf [3] have studied the temporal correlation of the intensity with respect to the angle for the co-polarised return. Qu and Dainty[5] have reported temporal correlation studies with respect to the angle (at and around backscatter) for both co- and cross-polarised detections. In this paper, we shall first summarise the temporal correlation properties. The photon counting statistics at different angles for both cases will also be reported.

The experimental setup was the same as in Ref.5. The experiments were carried out by using linearly polarised $515nm$ line Argon ion laser. The sample was 10% latex beads in water with average diameter $0.46\mu m$. The scattered light was detected by using a photomultiplier tube with a pinhole much smaller than the backscattered speckle size. The signals were then analyzed by a digital correlator and its associated micro-computer system.

2. TEMPORAL CORRELATION PROPERTIES

2.1. Auto-correlation function

The temporal correlation function measured in our experiments is defined as:

$$C(t) = \frac{< \delta I(0)\delta I(t) >}{< I >^2}$$

where the angular brackets represent an ensemble average. Typical sets of data for both co- and cross-polarised detections are plotted in Fig.2. It is found that the correlation curves are non-exponential due to multiple scattering. The resultant correlation curve is the superposition of the contributions from all different paths of different lengths.

For the co-polarised case the correlation decay is slower than that for the cross-polarised case. This reveals that apart from the involvement of single scattering for co-polarised correlation curves, the shorter paths contribute more, while for cross-polarised detection the longer paths have a larger role to play. Therefore, as shown later, in general the correlation time is longer for the co-polarised case than that for the cross-polarised case.

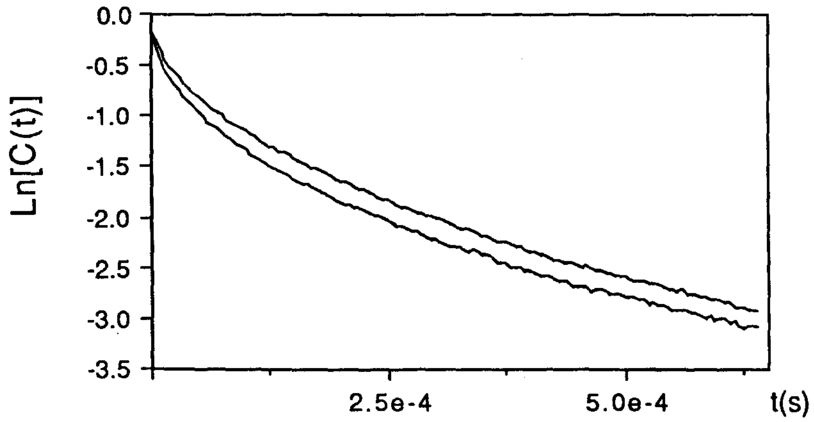

Fig.2. The logarithm of the temporal auto-correlations measured at backscatter for both co- and cross-polarised cases, with a sampling time of $5\mu s$. The speckle contrast $\frac{\sigma^2}{<I>^2} \approx 0.85$. The non-straight curves indicate the non-exponential decay of correlation functions. Co-pol.—upper curve, cross-pol.—lower curve.

2.2. Correlation time

To describe the non-exponential fluctuation rate and compare the different fluctuating behaviour at and near backscatter for both co- and cross-polarised cases, a correlation time is defined as follows:

$$\tau_c = \int_0^{+\infty} \frac{< \delta I(0)\delta I(\tau) >}{< (\delta I)^2 >} d\tau$$

The measurements of τ_c against angle for both co- and cross-polarised detection have been made and are shown in Fig.3. As can be seen from the figure, inside the enhancement cone, the temporal characteristics for co- and cross-polarised cases have very different behaviours. For the co-polarised case, the curve has a double-peak structure while a single-peak similar to that of the intensity enhancement appears for cross-polarised detection.

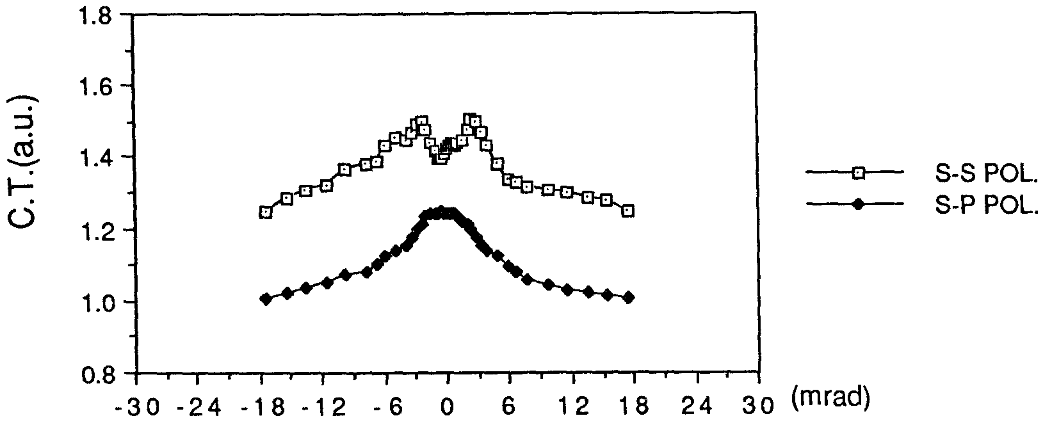

Fig.3. The measured correlation time against scattering angle for co- and cross-polarised detections

Here we give a physical explanation for the observed phenomena:

The different behaviour of the two cases arises from the fact that the contributions from the diffraction of the scattering paths and from the coherence contribution are weighted in terms of the measuring angle.

The correlation function of scattered light consists of contributions from two parts, namely, one from the incoherent intensity and the other from the coherent intensity. The contributions from incoherent light are uniform against angle for both co- and cross-polarised cases, while the coherent contributions result in different behaviours for co- and cross-polarised detections.

Let us now concentrate on the coherent part contributions to the correlation function. There are two fundamental factors among the coherent part contributions which have opposite effects on correlation time behaviour. One factor is from the contributions of the different light scattering paths of different lengths (path length contribution). The other factor accounts for the total coherent intensity contribution (enhancement factor contribution).

In the co-polarised case, the field correlation function is given by[3]

$$< E(0)E^{*}(\tau) >= \int_0^{+\infty} I(L)\exp(\frac{-L\tau}{4l^{*}\tau_o})dL$$

where $I(L)$ is the intensity contribution of transport paths of length L, l^* is transport mean free path and τ_o is a single scattering relaxation time.

It can be found from the formula that the contribution to the correlation from large L decays faster than that from shorter L, that is to say, the contribution from longer paths (larger L) have shorter correlation time, and shorter paths have longer correlation time. The resultant correlation time due to path length contribution depends on how the contributions from different paths are weighted.

As pointed out in Ref.2, when one moves away from the backscatter ($\theta = 0$), but still inside the enhancement cone, only paths of length $L < \frac{\lambda^2}{l\theta^2}$ (l is mean free path) contribute significantly to the intensity enhancement. Therefore when θ becomes larger, the upper limit of L becomes smaller, shorter paths contribute more and the correlation time due to this contribution therefore becomes longer.

When one moves away from backscatter, the time-reversed paths become out of phase. This makes the coherent intensity decrease and subsequently reduce the coherent enhancement contribution to the correlation time.

It can be seen by combining the contributions to the correlation time from mentioned two factors, that at backscatter the longest paths count more and hence the correlation is shorter here. When θ increases, as shorter

and shorter paths contribute more, the correlation time becomes larger but there comes to a point where the coherent enhancement contribution has decreased to such a low amount that the correlation time goes down to level the incoherent correlation time.

In the cross-polarised case, the coherent enhancement is mainly due to lower-order multiple-scattering process and longer paths do not contribute significantly, even at the backscatter. This can be justified from the fact that the enhancement factor for cross-polarised case is very low (typically 1.2). The enhancement factor would be larger if longer paths contributed because longer paths would give rise to a sharper, narrower and higher enhancement peak centred at backscatter.

So the conclusion is when one goes towards the backscatter right from the edge of the enhancement cone, slightly longer paths contribute more and more, and the long path contribution saturates. Therefore the correlation time behaviour due to this contribution is as follows: it decreases when θ decreases and levels off near backscatter because of the saturation effect. The enhancement contribution in cross-polarised case is the same as that in the co-polarised case. By superposing the above mentioned two factors, it is found that in cross-polarised detection the correlation time against angle appears to be a single peak structure in contrast of double-peak structure for co-polarised detection.

3. PHOTON COUNTING STATISTICS

The measurements of the photon counting distribution were made with a very small sampling pinhole ($\approx 30\mu rad$) and sampling time ($=10\mu s$) to avoid spatial and temporal averaging effects. The normalised factorial moments were also calculated from the measured data.

An experimental comparison between the probability distributions at backscatter ($\theta = 0$) and away from backscatter ($\theta \approx 12mrad$) was made. The different means at backscatter and a larger angle imply the enhanced backscattering.

Fig.4. Photon counting distribution P(n,T), sample time T=$10\mu s$, measured at $\theta = 0$ and $\theta = 12mrad$, with average photons per sample of $\approx$1.27 and $\approx$0.89 respectively.

To compare and study photon statistics at the enhancement peak and wings for both co- and cross-polarised cases, it is convenient to choose normalised factorial moments as the descriptive parameters. The m-th factorial moment is defined as:

$$\mathcal{F}(m) = \left\langle \frac{n!}{(n-m)!} \right\rangle = \sum_{n=0}^{\infty} n(n-1)\cdots(n+m-1)P(n,T) \tag{1}$$

The m-th normalised factorial moment is:

$$F(m) = \frac{\mathcal{F}(m)}{<n>^m} \tag{2}$$

where angular brackets represent an ensemble average.

Although the probability distributions at different angles in both co- and cross-polarised cases are different, the normalised factorial moments (F(m)s) at different angles are similar for both detecting polarisations. The measurements of these are given in Table 1. It is found, therefore, the normalised factorial moments are independent of angle and detecting polarisation. The intensity probability distribution does not vary with the angles and polarisations except for the different mean counting rates.

F(m)	Co-polarised		Cross-polarised	
	0(mrad)	12(mrad)	0(mrad)	12(mrad)
m=1	1.00	1.00	1.00	1.00
m=2	1.86	1.85	1.82	1.81
m=3	5.03	4.98	4.85	4.77
m=4	17.53	17.19	16.25	16.08
m=5	72.65	70.47	64.58	64.27

Tab.1. Comparisons of normalised factorial moments of co- and cross-polarised cases, at backscatter and 12*mrad*, showing the independence of the normalised factorial moments upon the detecting polarisations and angles near backscatter.

To explain the measured statistics, a comparison of measured photon statistics was made with Gaussian statistics for the field. For Gaussian speckle the intensity probability distribution has the form of negative exponential[6] $[P(I) = \frac{1}{\bar{I}}\exp(-\frac{I}{\bar{I}})]$ which corresponds to a Bose-Einstein[7] photon counting distribution $[P(n,T) = \frac{\bar{n}}{(1+\bar{n})^{1+n}}$, $\bar{n} = \alpha\bar{I}(T)]$. The comparison was done by comparing the measured normalised factorial moments with those of Bose-Einstein distribution [m!].

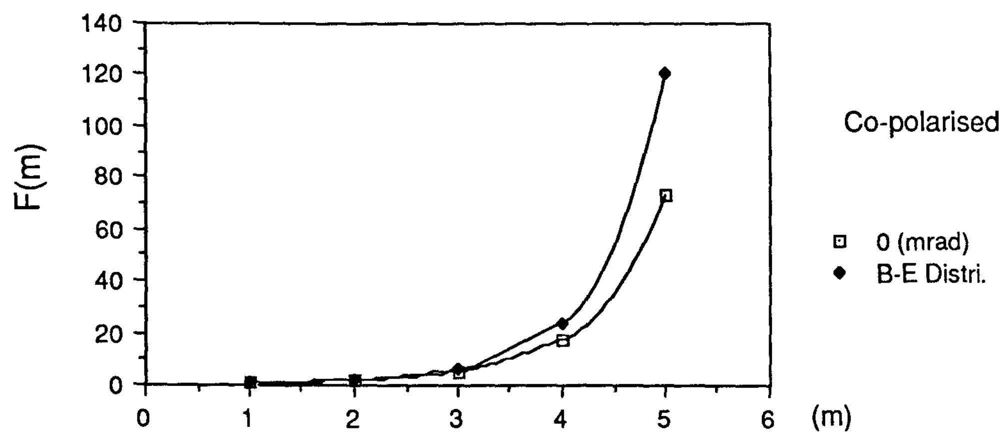

Fig.5. Measured normalised factorial moments at backscatter in co-polarised detection are compared with those of Bose-Einstein distribution.

Fig.5 shows that there is a departure between the measured statistics and those of Gaussian speckle. Kaveh *et al* [8] have suggested that the intensity distribution in such cases can be described by gamma distribution function

with its descriptive parameter u=2.5. Wolf *et al* [9] have also suggested the same distribution function but with u=1.3 for their case. Here we try to describe the measured distribution function still with gamma distribution function, and try to get value for u by fitting the measured normalised factorial moments with those having gamma distribution function with appropriate u value.

The gamma intensity distribution is as follows:

$$P(I) = \frac{1}{\bar{I}}\left(\frac{I}{\bar{I}}\right)^{u-1}\frac{u^u}{\Gamma(u)}\exp\left(\frac{-uI}{\bar{I}}\right) \tag{3}$$

where $\bar{I}$ is the mean intensity and u is a constant; as u tends to 1, this distribution tends to a negative exponential distribution function which is the intensity distribution shown by Gaussian speckle.

Here we derive and calculate the normalised factorial moments of photon counts when the detected intensity I obeys Gamma distribution function. By using Mandel's formula it is found the m-th factorial moment is proportional to the m-th moment of the intensity I,

$$\mathcal{F}(m) = \left\langle \frac{n!}{(n-m)!} \right\rangle = \alpha^m < I^m >$$

where α is the total quantum counting efficiency.

For the intensity distribution given by Eq.(3), the m-th moment of the intensity I is:

$$\begin{aligned} < I^m > &= \int_0^\infty I^m P(I) dI \\ &= \int_0^\infty I^m \frac{1}{\bar{I}}\left(\frac{I}{\bar{I}}\right)^{u-1}\frac{u^u}{\Gamma(u)}\exp\left(\frac{-uI}{\bar{I}}\right)dI \\ &= \frac{u^u}{\bar{I}^u\Gamma(u)}\int_0^\infty I^{u+m-1}\exp\left(\frac{-uI}{\bar{I}}\right)dI \\ &= \frac{\bar{I}^m}{u^m}\frac{\Gamma(u+m)}{\Gamma(u)} \end{aligned} \tag{4}$$

Therefore by noting $\alpha\bar{I} = \bar{n}$, the m-th factorial moment is:

$$\mathcal{F}(m) = \alpha^m < I^m > = \frac{\bar{n}^m}{u^m}\frac{\Gamma(u+m)}{\Gamma(u)} \tag{5}$$

So the m-th normalised factorial moment is:

$$\begin{aligned} F(m) = \frac{\mathcal{F}(m)}{\bar{n}^m} &= \frac{1}{u^m}\frac{\Gamma(u+m)}{\Gamma(u)} \\ &= \frac{1}{u^m}(u+m-1)(u+m-2)\cdots(u+1)u \end{aligned} \tag{6}$$

when u $\rightarrow$ 1, it tends to m! which corresponds to that of Bose-Einstein distribution.

A comparison between the experimental data and the calculation values from Eq.(6) was made and shown in Fig.6. After the dead time correction it is found that the best fit is when u=1.20.

At present, we can only speculate as to the cause of the discrepancy between the measured fluctuation (u=1.20) and that associated with a Bose-Einstein distribution (u=1). The temporal and spatial integration was small and the difference is not due to this effect. On the other hand, the limited sample size used to measure the photon counting distribution, typically on the order of 10^5 independent samples, may explain the discrepancy[10,11]. If the effect is real, it is possible that it may be accounted for by multiple scattering[12] although this is unlikely.

4. CONCLUSIONS

The temporal correlation behaviour of light scattered by a dense collection of random scatterers is strongly dependent on the polarisation of the scattered intensity. The statistics of the scattered intensity show a small deviation from those expected of Gaussian speckle and this is probabily due to experimental factors.

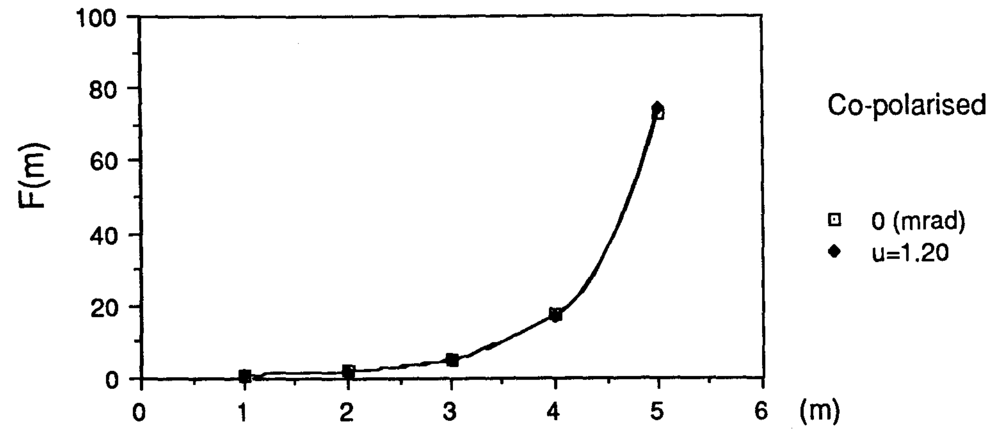

Fig.6. Measured normalised factorial moments are compared to the moments of a Gamma distribution with u=1.20.

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