Double Passage Imaging Through Turbulence

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ABSTRACT

Recent progress in our understanding of imaging when an object is both illuminated and viewed through the same random screen is reviewed, with particular emphasis on the use of non-redundant apertures.

1. INTRODUCTION

It is well known that the average image intensity $I(x,y)$ of an incoherently illuminated object of intensity $O(x,y)$ is blurred when viewed through turbulence. In the case of astronomical imaging, the loss of resolution compared to that determined by the diffraction limit may be 50 times or more. To overcome this limitation, techniques such as speckle interferometry[1], bispectral imaging[2], speckle holography[3] or adaptive optics[4] may be used.

The analysis of imaging through turbulence when the object is coherently (e.g. laser) illuminated is more complicated. In this case, one has to specify how the object is illuminated and whether the object is deterministic or statistical. As regards the illumination, the basic choices are either that the object is illuminated by a non-statistical wave (for example, a plane wave) or that it is illuminated by a statistical wave, for example, one that has already propagated through turbulence. When the object is illuminated by a plane wave, the effect of the turbulence on the average image intensity is the same as for the case of an incoherent object[5], although it should be noted that imaging in coherent light is non-linear in the intensity and therefore the actual image intensities are different in the two cases. In the case of illumination through turbulence, one has to decide whether the turbulence through which the object is viewed is the same as, or different from, the turbulence through which it was illuminated. An interesting case is when the object is illuminated and viewed through the same turbulence and we refer to this as "double passage" imaging.

There is an extensive literature on double passage effects in the propagation of light through turbulence[6–19]. The basic effect that is predicted (and experimentally observed) is that the intensity in the backscatter direction is significantly higher than in directions close to backscatter. This effect is closely related the the phenomenon of "enhanced backscattering" that is observed for light scattered by dense volume media[20] and rough surfaces[21].

The case of imaging when the object is illuminated and viewed through turbulence was examined in a general way by Fante[22] and for special cases in References 23–25. The key result is that the average image intensity contains diffraction limited information on the object structure, in contrast to the case of single passage imaging. For illumination and viewing through a normal filled aperture, the effect is weak — the signal is low and the noise is high[22,23]. But the effect may be quite strong for a non-redundant array aperture[24,25] and a deterministic object.

In this paper, the main results for double passage imaging are summarised. Section 2 gives a brief description of single passage imaging in incoherent and coherent illumination. The subject of double passage imaging is introduced in Section 3 using Michelson (two-slit) interferometry to explain the physics of the effect. Section 4 deals with double passage imaging with a filled aperture and Section 5 with non-redundant apertures. The final Section summarises the main results and speculates on possible future work.

2. SINGLE PASSAGE IMAGING

Consider the case of long-exposure imaging of an incoherently illuminated, or self-luminous, object through turbulence. If the turbulence can be represented by a random screen located at the pupil of the imaging system (this assumption will be applied throughout this paper), then the average Fourier transform of the image intensity, $< i(u,v) >$, is related to the Fourier transform of the object intensity, $o(u,v)$, by

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$$< i(u,v) > \quad = \quad o(u,v) \times T_{opt}(u,v) \times T_a(u,v), \qquad [1]$$

where $T_{opt}(u,v)$ is the optical transfer function of the imaging system (i.e. the autocorrelation of the pupil function) and $T_a(u,v)$ is the long exposure "atmospheric" transfer function; this is equal to the statistical autocorrelation of the complex amplitude $A(\xi,\eta)$ at the pupil due to the propagation of a plane wave through the turbulence,

$$T_a(u,v) \quad = \quad < A(\xi',\eta')A(\xi'+\xi,\eta'+\eta) > / < |A(\xi',\eta')|^2 >, \qquad [2]$$

where the pupil coordinates (ξ,η) are related to spatial frequencies (u,v) in the image by $\xi = \lambda f u$ and $\eta = \lambda f v$, where λ is the wavelength and f is the focal length. The form of the atmospheric transfer function is such that it is attenuated at characteristic spatial frequency $w = \sqrt{u^2+v^2} = r_0/\lambda f$, where r_0 is the so-called Fried parameter[26]; this should be compared to the diffraction limited cut-off frequency $w_{dl} = D/\lambda f$, where D is the telescope diameter and typically $D >> r_0$. Equation (1) can also be written in the form

$$< i(u,v) > \quad = \quad i_{dl}(u,v) \times T_a(u,v), \qquad [3]$$

where $i_{dl}(u,v)$ is the Fourier transform of the diffraction limited image intensity in the absence of the atmosphere (but including the effect of the telescope aperture).

If the object is illuminated by a (coherent) plane wave, then Eq.(3) still holds[5]; however, since the image intensity in the absence of the atmospheric turbulence is now the *coherent* image, the quantity $i_{dl}(u,v)$ differs from its incoherent counterpart. But the effect of the turbulence is the same as in the incoherent case — it strongly attenuates spatial frequencies greater than $r_0/\lambda f$ in the average Fourier transform of the image intensity.

3. DOUBLE PASSAGE — MICHELSON (TWO-SLIT) INTERFEROMETRY

At first glance, it might seem that an average image formed when the object is both illuminated and viewed through the same turbulence would also lose high spatial frequencies. In fact, there is diffraction-limited information in the average image intensity and the physical origin of this is most easily seen by considering the fringes formed by a Michelson (two-slit) interferometer viewing a point scatterer. Since image formation can always be regarded as an interferometric process involving many pairs of points within the pupil, an understanding of the formation of Michelson fringes illustrates the general case. In the following analysis, the turbulence is assumed to be represented by a random screen at the pupil of the imaging system. In order to get simple expressions for the average image intensity, this screen is often assumed to have complex gaussian statistics (thus permitting simple factorisation of the fourth order moment of the field) but this assumption is not believed to be of great physical significance. The assumption that the three-dimensional distribution of turbulence is compressed to a screen in the pupil, thus making the imaging isoplanatic, is probably a far greater practical limitation.

Consider the geometry shown in Figure 1. A point scatterer is illuminated through two slits A and B and the scattered light is collected by the two slits forming an interference pattern to the left of the slits. In the absence of the random screen, unit contrast fringes are formed as in Figure 2(a); the Fourier transform of these is given by the expression

$$i(u) \quad = \quad \delta(u) + 0.5\delta(u+\xi_0/\lambda f) + 0.5\delta(u-\xi_0/\lambda f), \qquad [4]$$

where the slit positions in the pupil are $\pm\xi_0$; that is, there are two side-bands of height 0.5 relative to the central peak.

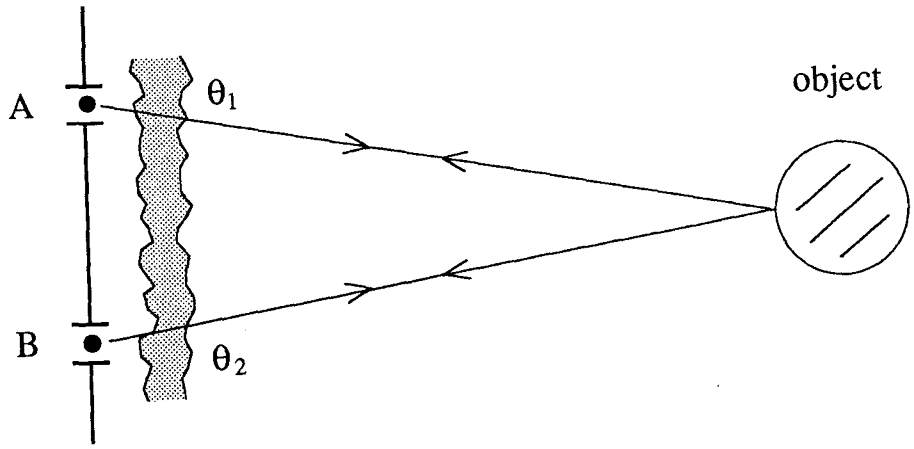


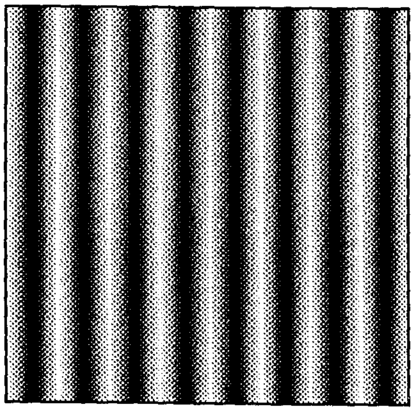
Figure 1

With the random screen present, single passage imaging does not give any fringe pattern on average if the phase fluctuation $\theta_1 - \theta_2 \gg 2\pi$ — instantaneously, of course, a fringe pattern is formed, but the position of the central fringe varies by more than one fringe period over time and on average the recorded intensity is uniform (i.e. a fringe pattern of zero visibility, with no side-lobes in its Fourier transform). A similar effect is obtained in double passage imaging through *different* screens, and this is shown in Figure 2(b).

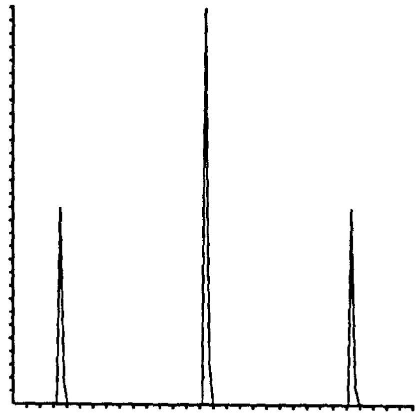
Consider now the case of double passage imaging through the *same* random screen and the four possible light paths AOA, BOB, AOB and BOA. The light paths AOA and BOB acquire phases of $2\theta_1$ and $2\theta_2$ respectively, giving a phase difference $2(\theta_1 - \theta_2)$ and therefore a uniform light intensity on the average. However, the phase acquired by the path AOB, $\theta_1 + \theta_2$, exactly equals that acquired by path BOA, $\theta_2 + \theta_1$, regardless of the fact that θ_1 and θ_2 may be random variables; since these two paths are the same, a stationary interference pattern is formed. For a pure phase screen, a straightforward analysis gives the following expression for the Fourier transform of the average image intensity[24]:

$$< i(u) > \; = \; \delta(u) + 0.25\delta(u + \xi_0/\lambda f) + 0.25\delta(u - \xi_0/\lambda f), \qquad [5]$$

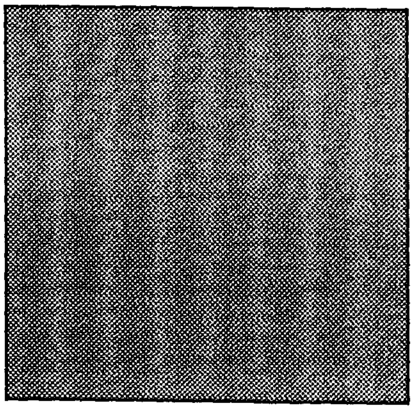
for the case in which the phases θ_1 and θ_2 are uncorrelated. It is clear from Eq.(5) that a fringe pattern is formed whose contrast is just one-half of the contrast produced when no random screen at all is present. Thus the double passage has produced diffraction-limited information in a situation where single passage does not. Physically, this is due to the fact that *forward and reverse scattering paths are coherent* and clearly this principle will apply to more general apertures.



(a) No random screen.

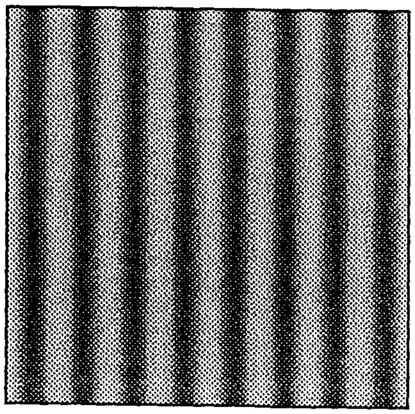


(d) Fourier Transform of (a).

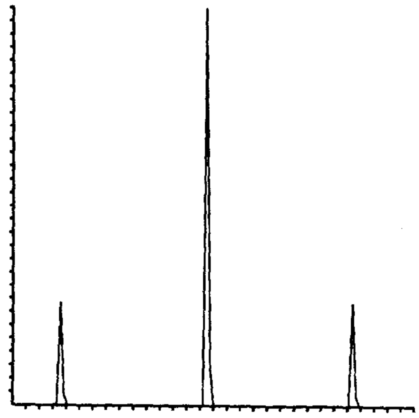


(b) Double passage - different screen.

(e) Fourier Transform of (b).



(c) Double passage through same screen.



(f) Fourier Transform of (c).

Figure 2

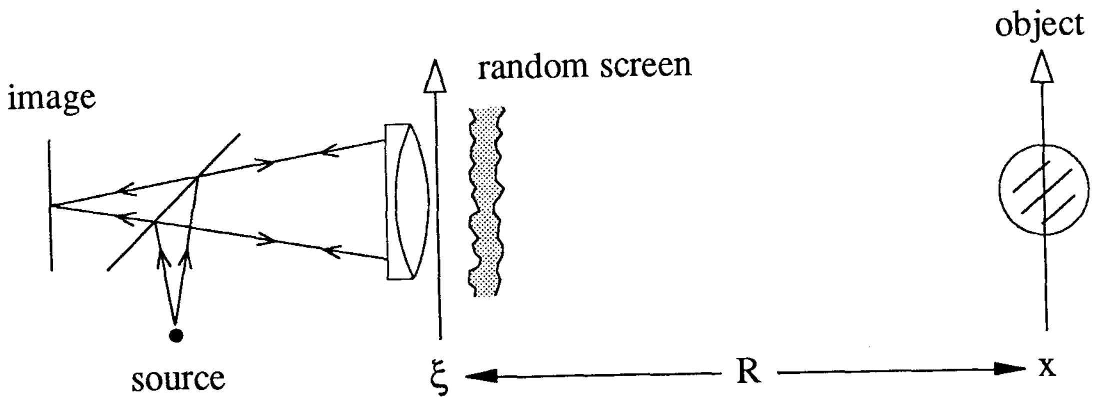


Figure 3

4. DOUBLE PASSAGE — FILLED APERTURE

Figure 3 shows the general geometry for filled and non-redundant aperture imaging. If it is assumed that the object is in the far-field of the illuminating (and viewing) aperture, and that the aperture is unobstructed, then the Fourier transform of the average image intensity can be shown[23] to be given by the expression

$$< i(u) > \;=\; K\, T_a(u)\left[\tilde{O}_w(u) \otimes T_a(u)\right] + \int_{-\infty}^{+\infty} P\Big(\frac{\xi}{D-|\lambda f u|}\Big)\left|o_a\Big(\frac{2\xi}{\lambda R}\Big)\right|^2 d\xi, \qquad [6]$$

where

$$o_a(u) \;=\; \int_{-\infty}^{+\infty} T_a(u')o^*(u'+u)\,du', \qquad [7]$$

K is a constant, $P(.)$ is the pupil function and $\tilde{O}_w(u)$, which is the weighted Fourier transform of the object intensity, is given by,

$$\tilde{O}_w(u) \;=\; \int_{-\infty}^{+\infty} OTF(u')o(u')o^*(u'+u)\,du', \qquad [8]$$

The important term is the second part of Eq.(6): this shows that the average Fourier transform of the image intensity is the weighted sum of values of the energy spectrum $\left|o_a\left(\frac{2\xi}{\lambda R}\right)\right|^2$ of a modified object spectrum (given by Eq.7). It

should be noted that the *highest* spatial frequency in the image contains information on the *zero* spatial frequency of the object and that progressively lower frequencies of the average image are cumulative integrals of higher and higher frequencies in the object; this complementary behaviour is characteristic of "tilted coherent illumination" and will also be encountered for the case of the non-redundant aperture. Inversion of Eq.(6) to give the energy spectrum of the object is obviously rather difficult.

5. DOUBLE PASSAGE — NON-REDUNDANT APERTURE

The basic property of double passage imaging that allows diffraction limited information to be transmitted in the average image was illustrated in §3 — the coherence of forward and reverse scattering paths provides the diffraction limited information. In the filled aperture, for every forward and reverse pair of paths of a certain separation in the pupil, there are many others of the same separation that add incoherently and greatly reduce the signal (i.e. the contribution of the coherent pair). This dilution of the signal can be significantly reduced using a non-redundant aperture for illumination and viewing[24,25].

We represent a non-redundant aperture as an array of N pinholes such that the vector separation between any two is unique. The transmission of such an aperture is denoted (in one dimensional notation) by

$$P(\xi) = \sum_{j=1}^{N} \delta(\xi - \xi_j). \qquad [9]$$

If the object is in the far-field of the aperture, and if the transmission of the random screen is uncorrelated for all pairs of pinholes, then the Fourier transform of the average image intensity reduces to a particularly simple form[24]:

$$< i(u) > |_{u=(\xi_m - \xi_l)/\lambda f} = |o([\xi_m + \xi_l]/\lambda f)|^2 \quad \delta(u - [\xi_m + \xi_l]/\lambda f), \qquad [10]$$

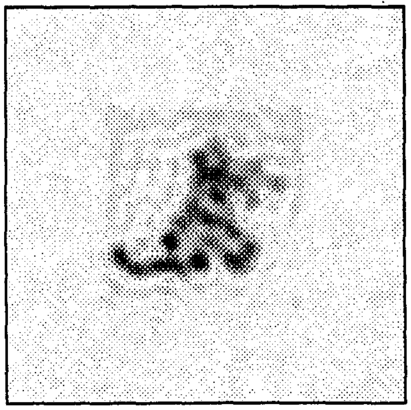
where l and m are indices over all N pinhole locations and $l \neq m$. Equation (10) describes a simple mapping between frequencies $(\xi_m - \xi_l)/\lambda f$ in the average image and frequncies $(\xi_m + \xi_l)/\lambda f$ in the object energy spectrum, and is the non-redundant aperture analogue of Eq.(6) for the completely filled aperture. However, for the case of the non-redundant aperture, it is clearly trivial to recover the energy spectrum of the object from the spectrum of the average image intensity.

The results presented above apply to an infinite (ensemble) average of the image intensity. The effect of finite averaging is to introduce an object dependent bias into the measured data which complicates matters considerably[25]. This bias is minimised for just two apertures (i.e. the Michelson case).

6. SUMMARY AND FUTURE WORK

We have shown that, for a deterministic object, double passage imaging can lead to the retention of diffraction limited information in the average image intensity. Recovery of this information is difficult for the filled aperture but is straightforward for either a two-pinhole or a non-redundant array of pinholes.

Equations (6) and (10) indicate that only the energy spectrum of the object can be estimated in double passage imaging. Future work should concentrate on the possibility of recovering complete images. Figure 4 is a computer simulation that illustrates our first step in this direction[25]; Fig. 4(a) is a diffraction limited image of an object (a character from a famous childrens' story) taken with a filled aperture and no atmosphere — it represents the best possible reconstruction. Figure 4(b) is a long exposure image taken by illuminating through one screen and viewing through an uncorrelated one. Figure 4(c) is a double passage image, reconstructed by sequentially covering the central (lower) frequencies with two-pinhole apertures and then using a phase retrieval algorithm to reconstruct the object from its Fourier magnitude; although the reconstruction is far from perfect, it represents a considerable improvement on Fig. 4(b).



(a) Diffraction-limited image.

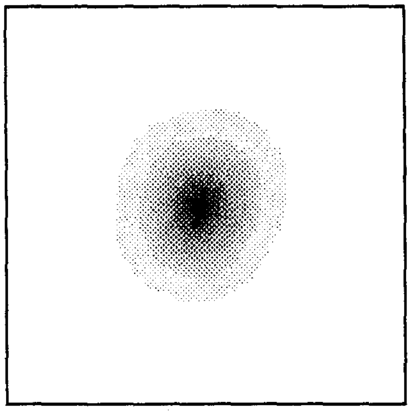
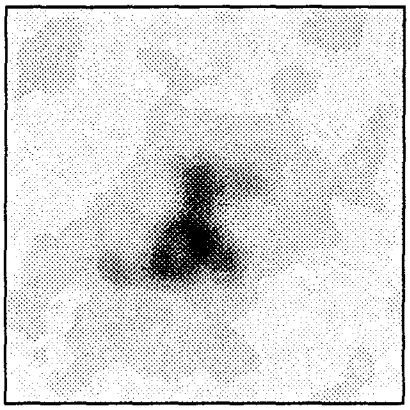


Figure 4

(b) Image through uncorrelated screens.



(c) Double passage Image.

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