Experimental study of enhanced backscattering from one- and two-dimensional random rough surfaces

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An experimental study of backscatter enhancement from rough surfaces is presented. The Stokes parameters of the average scattered light from two-dimensional rough surfaces show the presence of an unpolarized component, which lends support to the multiply scattering ray model. Experimental data from one-dimensional rough surfaces are compared with numerical calculation.

INTRODUCTION

The phenomenon of light scattering from rough surfaces (random or otherwise) has attracted much attention, both experimentally and theoretically. This subject is of particular importance in areas that involve using a wave, either acoustic or electromagnetic, as a probe to observe material and surface properties, e.g., interpretation of radar returns (from the surface of the Earth as well as from other planetary bodies) and noncontact surface characterization.

Recently the enhancement of scattered light intensity in the backscatter direction from metallic rough surfaces was reported by Mendez and O'Donnell. This backscatter peak from other random rough surfaces had been previously observed; a sudden increase in the brightness of the Moon when it approached its fullness was reported as far back as 1924 by Markov, while Oetking reported this effect to be present when scattered light from rocks, as well as certain reference samples, was observed. This backscatter peak, also called the opposition effect in the literature, has been reported by several other authors; the peaks usually have a small angular width (typically 2 to 3 deg) and result when the scattering of light occurs in the volume as well as on the surface. The phenomenon of backscatter enhancement as observed by Mendez and O'Donnell differed from what the other authors reported in that they used metallic, high-sloped, single-scale, Gaussian, random rough surfaces, whose standard deviation of surface height was much larger than the incident wavelength. The key property was that the scattering of light was confined to the surface owing to its metallic nature. This effect was normally accompanied by a large cross-polarized component.

The high-sloped nature of the surface meant that multiple scattering was a significant contribution to the scattered light. The methods that can be used to explain this phenomenon analytically are limited because of the restrictions imposed on the available scattering theories. Physical optics cannot be used since it accounts only for single scattering and only when the surface structures are much larger than the incident wavelength. Analytical multiple-scattering theories utilizing the extended boundary condition are expressed in a perturbation series, and, owing to the difficulty in calculating high-order terms and the slowness or complete lack of convergence, they have been limited to the case of low-sloped surfaces. The full-wave solution may account for the enhanced backscatter peak while single scattering only is used.

The technique of numerical calculation of the scattered light has been available for some time. This method is computationally highly intensive, and hence the application of this method was quite limited until recently. Perfectly conducting surfaces were normally considered, and the intent was largely to establish the range of validity of the available scattering theories such as the physical-optics solution and the full-wave theory. The technique has also been used to calculate the scattered light from high-sloped surfaces, and enhanced backscatter peaks were observed in the calculated values; such effects were also observed when real metallic surfaces were considered. The major flaw of this procedure is that it gives little physical insight into the scattering process; all effects such as shadowing, multiple scattering, and light–surface interaction are mixed inseparably together.

Mendez and O'Donnell proposed a simple model involving multiple scattering of rays from surface facets; it was an analog of that used in the volume-scattering case. Jake- man used a model consisting of a deep random phase screen with a mirror placed just behind it. However, although these models explained qualitatively the presence of enhanced backscatter peaks, they leave much to be desired, e.g., they do not take into account the polarization of the scattered light or predict the detailed shape of the scatter envelope.

The research reported here is an extension of that presented by Mendez and O'Donnell. The normalization method used to scale the experimental data will be discussed briefly. The polarization behavior of the scattered light from two-dimensional random rough surfaces that exhibit enhanced backscatter peaks is discussed. Interpretation of Stokes parameters leads to an alternative way of mapping the scattered light; instead of the usual copolarized and cross-polarized intensity one can plot the polarized and unpolarized components. In this context, unpolarized means that there is no preferred direction of polarization over the solid angle.
of measurement (which encompasses many speckles). Finally, an experimental study of approximately one-dimensional random rough surfaces is presented, and the experimental data are compared with numerically calculated values where possible.

**SCATTEROMETER RESPONSE**

The name scatterometer designates the equipment\(^{21}\) that was used to perform experiments involving the measurement of scattered light as a function of the angle of incidence \(\theta_i\) and the scattering angle \(\theta\).

Figure 1 is a schematic diagram of the scatterometer viewed from above. Two laser light sources were available: He–Ne (Spectra-Physics 105-1 laser, wavelength \(\lambda = 0.633 \text{ \mu m}\)) and CO\(_2\) (Edinburgh Instruments WL-4, \(\lambda = 10.6 \text{ \mu m}\)). The spot size at the sample was approximately 10 mm in diameter, and the incident beam was collimated. The sample mount held the rough surface such that the mean surface normal was horizontal, although it was possible to tilt the surface normal slightly off the horizontal plane. The sample mount and the rotating arm were movable, both having the same rotational axis, each controlled by an individual stepper motor. The values of angles were such that \(\theta_i\) was measured clockwise from the surface normal and \(\theta\) anticlockwise from the surface normal, ensuring that \(\theta = \theta_i\) in the specular direction. The detectors used were a Hamamatsu R647 photomultiplier for the visible light and a Plessey PLT222 pyroelectric detector for the far-infrared radiation. When performing experiments in the far infrared, we used a chopper in conjunction with a phase lock-in amplifier (Stanford Research SR530) to eliminate the background noise, the incident beam being chopped at typically 80 Hz. A microscope objective of 5-mm diameter and a CdS lens of 1-cm diameter were used as integrating lenses for the visible and the far infrared, respectively. The distance between the rotational axis and the integrating lens was 62 cm; the angular resolution of the measurement of the scattered light was thus approximately 0.5° for the visible and 1.0° for the infrared.

The light incident upon the rough surfaces was always coherent, and hence a speckle pattern was generated. The detector response \(R_d\) is proportional to the spatial integral of speckles in the solid angle of the integrating lens, i.e.,

\[
R_d = \mathcal{R} \int_{\Delta \Omega} d\Omega W(\Omega) J(\Omega),
\]

where \(\Delta \Omega\) is the solid angle of the integrating lens, \(W(\Omega)\) a weighting function, \(\mathcal{R}\) is the constant of proportionality, and \(J\) is the radiant power. Here we make an assumption that the finite spatial average is equal to an ensemble average, i.e.,

\[
R_d = \mathcal{R} \langle J \rangle (\Delta \Omega'),
\]

where \(\Delta \Omega'\) is the effective integration angle and does not necessarily stay constant for all \(\theta\). \(\langle J \rangle\) denotes the average of the radiant intensity for that particular scattering angle, for an ensemble of statistically identical but independent rough surfaces. We introduce a new quantity, the mean normalized differential scattering cross section (DSCS) \(\Sigma\), defined as

![Fig. 1. Schematic diagram of the scatterometer viewed from above. PMT, photomultiplier tube.](image)

![Fig. 2. Scatter envelope from a MgO surface, s incident polarization (\(\lambda = 0.633 \text{ \mu m}\)). Second curves from bottom denote \(\Sigma_{\text{tot.}}\); second curves from bottom denote \(\Sigma_{\text{pp.}}\); topmost curves denote \(\Sigma_{\text{scat.}}\). \(\theta_i\) is a, 0°; b, -30°, and c, -60°. The solid curves denote the case of a perfect Lambertian surface with a perfect scatterometer response.](image)
\[ \Sigma = \frac{\langle I \rangle}{\Phi_i}, \]  

(3)

where \( \Phi_i \) is the incident power. \( \Sigma \) is related to the well-known bidirectional reflection function \( f(r) \) by a simple expression:

\[ \Sigma = \langle f(r) \rangle \cos \theta. \]  

(4)

To denote the polarization property of the incident and scattered light, subscripts are appended to \( \Sigma \); the first letter of the subscript denotes the polarization state of the incident light, and the second the polarization state of the detected light; e.g., \( \Sigma_{sp} \) is the DSCS for the case when incident light is s polarized and the detected light \( p \) polarized.

Hence, by using Eqs. (2) and (3), an accurate value of \( \Sigma \) can be obtained once the angular dependence of \( A\Omega' \) is known, and this is usually done by the reference sample method.\(^{21}\) Normally, a Lambertian scatterer is sought as an ideal reference sample. If the directional reflectance (total scattered power divided by the incident power) is unity, the mean normalized DSCS for a Lambertian diffuser is given by a simple expression:

\[ \Sigma(\theta_i, \theta) = \frac{1}{\pi} \cos \theta. \]  

(5)

The problem with this method is that a perfect Lambertian diffuser is impossible to realize. Historically, a freshly smoked magnesium oxide (MgO) surface was used as an approximation to a Lambertian scatterer, although a barium sulfate \((\text{BaSO}_4)\) surface as made by the method prescribed by Eastman Kodak has established itself as a standard.\(^{24}\)

To check the response, \( \Sigma \) was initially obtained, for the case of a MgO surface, assuming that \( A\Omega' \) remains constant for all \( \theta \). The directional reflectance was assumed to be unity.

Figures 2a, 2b, and 2c show the scattering data from a MgO surface for \( \theta_i = 0^\circ, -30^\circ, \) and \(-60^\circ\), respectively. The incident light was \( s \) polarized, of wavelength \( \lambda = 0.633 \) \( \mu \)m. In each figure the copolarized component \( \Sigma_{css} \) and the cross-polarized component \( \Sigma_{csp} \) are shown together with the sum of both, i.e., \( \Sigma_{tot} \). Figures 3a, 3b, and 3c show similar data but with the incident polarization being \( p \) polarized. The fit with the cosine line is fairly good for both incident polarizations except for the presence of the backscatter peaks (cf. Ref. 6 and references therein).

Since our scatterometer gave an approximately cosine response for the case of an approximately Lambertian scatterer, it was decided that the assumption that \( A\Omega' \) remains constant for all \( \theta \) was a reasonable one to take. This approximation is implicit from this point onward. Another important approximation used was that, on normalization, the metallic surfaces were all assumed to be perfect conductors.
STOKES PARAMETERS OF SCATTERED LIGHT

Plate #313 was a gold-coated metallic two-dimensional Gaussian random rough surface. It was made by the method described by Gray,\textsuperscript{25} that is, by multiply exposing a photore sist-coated substrate to laser speckle patterns. After extensive analysis of the surface profiles obtained from a Taly surf profilometer with a sufficiently small stylus tip, it was found that plate #313 had a standard deviation of surface height $\sigma_h = 1.0 \pm 0.1 \mu m$ and a 1/e correlation length $\tau = 2.9 \pm 0.2 \mu m$, i.e., the surface structure was larger than the visible wavelength, but the standard deviation of the surface slope was high, approximately 0.5. This surface showed the enhanced backscatter peak as well as a large cross-polarized component. In fact, the scatter envelope was similar to that obtained from plate #83 in Ref. 2.

Stokes parameters provide a complete description of the polarization state of the scattered light. Ideally, they should have been obtained for all $\theta$ and for all angles of incidence. As such, they were calculated for normal incidence and for a few fixed scattering angles $\theta$ (one near the backscatter direction). The incident light was linearly polarized. The values of the Stokes parameters showed that for those fixed angles, the scattered light was composed of two components, at least to within the experimental error (5% of the first Stokes parameter): one that is linearly polarized in the direction of the incident light and another that is unpolarized (see below). For scattering of monochromatic light from a single surface realization, light scattered in a single direction will be completely polarized. However, what we are measuring is the scattered light integrated over a detector solid angle, and the measurement indicates is that there is a component of scattered light that, although com-
Fig. 8. Plate #440, s incident polarization ($\lambda = 0.633 \mu m$), for $\theta_i = a, 0^\circ; b, -10^\circ; c, -20^\circ; d, -40^\circ$. The thicker curves denote experimental $\Sigma_{pp}$, the thinner curves show the numerically calculated values. The measured cross-polarized component, denoted by the line almost coincident with the x axis, is shown only for the case of normal incidence.

Fig. 9. Plate #440, p incident polarization ($\lambda = 0.633 \mu m$), for $\theta_i = a, 0^\circ; b, -10^\circ; c, -20^\circ; d, -40^\circ$. The thicker curves denote experimental $\Sigma_{pp}$, the thinner curves show the numerically calculated values. The measured cross-polarized component, denoted by the line almost coincident with the x axis, is shown only for the case of normal incidence.
Figure 4a shows the plot of mean normalized DSCS $\Sigma_{\text{st}}$ and $\Sigma_{\text{sp}}$, while Fig. 4b shows the plot of mean normalized DSCS converted in the manner given by Eqs. (6), both for the case of plate #313, normal incidence, the incident wavelength's being $\lambda = 0.633 \, \mu\text{m}$. Figures 5, 6, and 7 show the similar quantities but for angles of incidence of $-10^\circ$, $-20^\circ$, and $-40^\circ$, respectively. It is interesting to note that the enhanced backscatter peak is confined mainly to the unpolarized component.

Multiply scattering rays, according to the model proposed by Mendez and O'Donnell, are the cause of depolarization as well as of enhanced backscatter peaks, i.e., they will not have a preferred polarization direction. Singly scattered rays have a preferred polarization direction. Hence the unpolarized component will contain the enhanced backscatter peak, according to this simple model. This is the case when one studies Figs. 4-7.

$\Sigma_{\text{unpol}} = 2\Sigma_{\text{sp}}$  \hspace{1cm} (6b)

\[ \Sigma_{\text{pol}} = \Sigma_{\text{st}} - \Sigma_{\text{sp}} \]  \hspace{1cm} (6a)

Figure 10. Plate #440, s incident polarization ($\lambda = 10.6 \, \mu\text{m}$), for $\theta_i = a, 0^\circ; b, -30^\circ; c, -50^\circ$. Labels as in Fig. 9.

Fig. 11. Plate #440, p incident polarization ($\lambda = 10.6 \, \mu\text{m}$), for $\theta_i = a, 0^\circ; b, -30^\circ; c, -50^\circ$. Labels as in Fig. 9.
SCATTERING FROM ONE-DIMENSIONAL SURFACES

Consider a surface whose surface height variation depends on one Cartesian coordinate, i.e., a one-dimensional surface \( z = h(x) \). If the incident electromagnetic field has either the electric or the magnetic field lying perpendicular to the incident plane, in this case the \( xz \) plane, there is no cross-polarized component, and the scattering problem reduces to a scalar-wave situation. The problem still remains intractable analytically, but it can be solved exactly by numerical calculation, which involves averaging the calculated scattered intensity over an ensemble of surface realizations.

Approximately one-dimensional random surfaces were made by etching a photoresist-coated substrate with a speckle pattern whose correlation length was much longer in one dimension than the orthogonal one. Two surfaces, both gold coated, were considered: plate \#440 (\( \sigma_h = 1.2 \pm 0.1 \mu m, \tau = 2.9 \pm 0.2 \mu m \)) and plate \#436 (\( \sigma_h = 1.6 \pm 0.1 \mu m, \tau = 5.2 \pm 0.3 \mu m \)). Surface parameters were obtained by using the traces measured in the \( x \) direction.

Following the method described in Refs. 15 and 16, numerical calculations were done for the case of a perfect conductor only. To ensure the accuracy of the solution obtained, the lengths of the discrete surface (40X) and the sampling distance (0.13X) were chosen such that the fluctuation of the normalized total scattered energy was less than 3%.

Figures 8a, 8b, 8c, and 8d show \( \Sigma_{se} \) for \( \theta_i = 0^\circ, -10^\circ, -20^\circ, \) and \(-40^\circ\), respectively, for the case of plate \#440, the incident wavelength being \( \lambda = 0.633 \mu m \). Figures 9a–9d show \( \Sigma_{pp} \) for the same angles of incidence. Negligible cross-polarized components were observed. The numerically calculated values, obtained after averaging over 400 surface realizations, are shown as solid curves. Enhanced backscatter peaks are observed, both in the experimental data and in the numerically calculated values. Agreement is good for small \( \theta_i \) but fails when \( \theta_i \) becomes large. The reason for this failure is not clear; it may be the fact that the finite conductivity of the real surface has not been taken into account, that the length of the surface in the computer calculation is too short, or that the condition that states that the total scattered power has to equal the incident power is not enough to
Figures 10a, 10b, and 10c show $\Sigma_d$ for $\theta_i = 0^\circ$, $-30^\circ$, and $-50^\circ$ respectively. Figures 11a-11c show $\Sigma_{pp}$ for the same angles of incidence. We believe that small cross-polarized components shown in Figs. 10a and 11a are due to the intrinsic error in the half-wave plate used in the scatterometer and the imperfect linear polarization state of the output beam from the CO$_2$ laser. Numerically calculated values are shown for comparison. In both experimental data and numerically calculated values, the coherent component has been removed, but in normalizing the experimental data its power was taken into account.

When one studies Fig. 12, which plots the normalized coherent component power for $s$ and $p$ polarization, experimental data, and calculated values, over a range of $\theta_i$.

Figures 13a, 13b, 13c, and 13d show $\Sigma_d$ and $\Sigma_{pp}$ for plate #436, with $\theta_i = 0^\circ$, $-10^\circ$, $-20^\circ$, and $-40^\circ$, respectively, for $\lambda = 0.633$ $\mu$m. Figures 14a, 14b, and 14c show $\Sigma_d$ and $\Sigma_{pp}$ for the same plate, with $\theta_i = 0^\circ$, $-30^\circ$, and $-50^\circ$, respectively, for $\lambda = 10.6$ $\mu$m. Numerical calculations were not possible for the case of plate #436, as the required array sizes were too large to handle. It is interesting to note that, unlike for plate #440, there seems to be little qualitative difference in the scatter envelopes resulting from the different incident polarizations. The key feature to note is the small or nonexistent enhancement in the backscatter direction, presumably a result of the small value of the average of the absolute slope.

CONCLUSION

Stokes parameters of the scatter envelope, which shows an enhanced backscatter peak, have been studied; it was found that the scattered light was composed of polarized and unpolarized light only and that the backscatter peak was confined to the unpolarized component. This result supports the hypothesis claiming that the backscatter peak is due to multiple scattering.

An experimental study of scattering of light from approximately one-dimensional surfaces was presented. Comparison with numerically calculated values was done wherever possible. There are disagreements in some cases, and the reasons are unclear at this stage of investigation. Although numerical calculation gives the numbers with which to compare the experimental data, further work needs be done if we are to understand the physical mechanism behind the scattering of light from high-sloped surfaces. It is hoped that the experimental results presented here will encourage such work.

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REFERENCES

27. Ref. 7, Chap. 6.