Measurements of light scattering by a characterized random rough surface

J C Dainty, N C Bruce and A J Sant
Blackett Laboratory, Imperial College, London SW7 2BZ, UK

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Abstract. Measurements are presented of the angular distribution of four wavelengths of light scattered by a one-dimensional random rough surface, whose probability density function is Gaussian with a standard deviation $\sigma = 1.22 \pm 0.02 \mu m$ and whose lateral correlation function is also Gaussian with $1/e$ width $\tau = 3.17 \pm 0.07 \mu m$. The wavelengths used are 0.63, 1.15, 3.39 and 10.6 $\mu m$. The surface is used in two forms: coated with gold and as an almost lossless dielectric. The results are compared to those predicted by a double scattering form of the Kirchhoff formulation. Agreement is good at small angles of incidence but less good at larger angles of incidence.

1. Introduction

The experimental observation of enhanced backscattering from random rough surfaces of large root-mean-square slope, first reported by Mendez and O’Donnell [1,2], has stimulated a re-examination of the problem of light scattering in the past few years. The main progress to date has been the development of ‘exact’ numerical codes for the solution of Maxwell’s equations from a one-dimensional surface illuminated with either $s$ (i.e. TE) polarization or $p$ (i.e. TM) polarization [3–7]. With $s$ polarization, the electric vector is parallel to the grooves, whereas with $p$ polarization it is perpendicular to the grooves, as in figure 1 (this figure also shows the sign convention used for the incident and scattering angles). An important feature of the work of Mendez and O’Donnell was that the surfaces were relatively well characterized, with Gaussian statistics for the surface height and a single-scale Gaussian correlation function. Since the statistics of the surface were known, a critical comparison between experiment and theory could be made with confidence.

The shape of the scattering cross section curves with angle of observation for high-sloped surfaces is quite different from that for simple low-sloped ones and early numerical results [3] were encouraging since they were in fairly good agreement with the experimental ones particularly at near-normal incidence. In order to carry out a more critical comparison between real experiments and numerical ones, it is important that the surface is well characterized and also helpful if a range of wavelengths are used. The results presented here are intended to supplement those already reported [8–10] with the aim of providing a reliable body of experimental data for comparison with numerical work and analytical theory. The surface used is one-dimensional, for two reasons: firstly, it can be characterized much more accurately than a two-dimensional one, since a sharp chisel-shaped stylus can be used in a surface profilometer; secondly,
Figure 1. Polarization and angle notation for in-plane scattering from a one-dimensional rough surface.

‘exact’ numerical calculations of the light scattering are only feasible at the present time for the one-dimensional case.

When comparing experimental measurements of light scattering with numerical computations, it is helpful if the numerical results provide some physical insight to the scattering process. For example, it is believed from the experiments that the mechanism giving rise to the enhanced backscatter peak is multiple scattering; however, numerical calculations based on exact theory do not separate the single and multiple scatter terms, and therefore do not provide the insight that is desirable (however, iterative solutions do separate the single and multiple scatter terms). For this reason, we have written numerical code based on a multiple (double and triple) scattering extension of the Kirchhoff boundary condition, including the effects of shadowing (see [11] for details and further references). In section 3 of this paper we compare the results of this code with the experimental results and ‘exact’ numerical code.

2. Experimental results

Master surfaces are produced by exposing a thick layer of photoresist (≈ 12 μm of Shipley S1400-37) to several statistically independent laser speckle patterns. Two versions of the surface were prepared using a replication technique [8], one being coated with ≈1000 Å of gold and the other being an almost lossless dielectric of refractive index \( n = 1.41 \) (at \( \lambda = 0.63 \) μm). Figure 2 shows the probability histogram of surface height and surface autocorrelation function, as measured by a Talystep profilometer whose stylus is a pyramid of 70° apex angle truncated by a flat region of ≈0.5 μm. Both are good fits to Gaussian functions, with the root-mean-square height \( \sigma = 1.22 \pm 0.02 \) μm and 1/e correlation length \( \tau = 3.17 \pm 0.07 \) μm. The angular distribution of the scattered
light was measured at four wavelengths (0.63 µm, 1.15 µm, 3.39 µm and 10.6 µm) using the equipment described in [10]. For each angle of incidence, measurements are made with p-polarization incident and p-polarization collected (‘p-p’ scattering) and s-polarization incident and s-polarization collected (‘s-s’ scattering); no crossed polarized components were detectable. For a perfect conductor, these measurements give a complete description of the scattering characteristics of the surface, but in general four scattering coefficients are required for materials of finite conductivity; these can be found by measuring the polarization of the scattered light for various input polarizations. Also, the measurements reported here yield the relative scattering cross section, as no absolute calibration is performed.

The relative scattering cross sections for angles of incidence of $0^\circ$, $-30^\circ$ and $-60^\circ$ and the four wavelengths are shown in figures 3 and 4 for the gold-coated surface and figures 5 and 6 for the dielectric surface. The enhanced backscatter peak, where present, occurs on the right-hand side of the graphs (i.e. at positive angles, see figure 1 for the sign convention for the angles) and any specular component is on the left-hand side (i.e. negative angles); for the 10.6 µm measurements, the specular peak was very much greater than the diffuse component and is not shown. A few features are of particular note.
Figure 3. Relative scattering cross section as a function of scattering angle for the gold-coated surface, for angles of incidence of $0^\circ$, $-30^\circ$ and $-60^\circ$, for p-p scattering (open circles) and s-s scattering (crosses). The left-hand column is for a wavelength of 0.63 $\mu$m, for which $\sigma/\lambda = 1.93$ and $\tau/\lambda = 5.02$ and the right-hand column for $\lambda = 1.15 \mu$m, for which $\sigma/\lambda = 1.07$ and $\tau/\lambda = 2.76$. The enhanced backscatter peak, where present, occurs at positive angles (right-hand side of each graph).

1) The enhanced backscatter peak and sidelobe structure are clearly visible for the shorter wavelengths at an angle of incidence less than approximately $-30^\circ$ for the gold-coated surface; the width of the peak is proportional to the wavelength. The peak is not observed for the p-p scattering at 10.6 $\mu$m for the gold surface or for scattering from the dielectric.

2) With the exception of the p-p case at 10.6 $\mu$m, the p-p and s-s scattering by the gold surface are very similar; for the dielectric surface, however, the p-p and s-s scattering cross sections are quite different, as one might expect by analogy with reflection from a planar surface. Using a value of $n = 1.41$ for the refractive index of the (almost lossless)
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Figure 4. As for figure 3 but wavelengths of 3.39 \( \mu \text{m} \) (\( \sigma/\lambda = 0.36, \tau/\lambda = 0.94 \)) and 10.6 \( \mu \text{m} \) (\( \sigma/\lambda = 0.12, \tau/\lambda = 0.30 \)). For the 10.6 \( \mu \text{m} \) curves, the angles of incidence were 0°, −20° and −40°. The (strong) specular component in the 10.6 \( \mu \text{m} \) curves is not shown.

dielectric gives a Brewster angle of \( \approx 55^\circ \). Considering single scattering to be the dominant mechanism and treating this as a reflection from a locally plane surface gives an expected minimum of the p-p scattered intensity at an angle equal to approximately (−110° − incident angle): the angles are roughly in accordance with this simple picture. The s-s and p-p scattered intensities in the backscatter direction appear to be almost equal to each other for all angles of incidence and wavelengths, for the dielectric.

(3) The overall shape of the curves is dramatically different from the Gaussian-type shapes (centred on the specular angle) normally encountered in scattering from low-sloped surfaces.

The principal purpose of figures 3 to 6 is to provide a reliable set of data for comparison with numerical calculations, and analytical theories should any become available.
Figure 5. As for figure 3 but for the dielectric surface, \( n_{0.65} = 1.41, n_{1.15} = 1.40 \). The vertical scale is not the same as that used in figure 3 (both are relative scattering cross sections).

3. Kirchhoff multiple scatter approximation

One can compare the above experimental results to those of 'exact' numerical calculations based on the extinction theorem and its extensions [3–7], and some comparisons of experiment and calculations for a perfect conductor were given in [10]. Although such comparisons are valuable, one problem with the 'exact' numerical solution is that it gives little physical insight into the problem. We have therefore attempted to extend the Kirchhoff approximation (i.e. tangent plane approximation for each scattering event) to double (and multiple) scattering [11].

The numerical calculations were carried out using the method described in [11] for a perfect conductor; typically the energy conservation (unitarity) held to better than 3% considering just the single and double scatter terms for surface # 46 (except for the
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Figure 6. As for figure 4 but for the dielectric surface, $n_{3.39} = n_{10.6} = 1.41$.

-60° results for which the departure from unitarity is $\approx 6\%$ and the results are averaged over approximately $10^3$ realizations. Figures 7 and 8 show the results of the calculations for $\lambda = 0.63 \text{ } \mu\text{m}$ and $1.15 \text{ } \mu\text{m}$ respectively, for incident angles of 0°, -30° and -60° and s-s and p-p scattering. Each graph shows the single, double and total scattered intensity. The enhanced backscatter peak occurs only in the double scatter component, showing conclusively that the enhancement is a multiple scattering effect. The enhancement is on the order of a factor of two in the double scattered component for all angles of incidence, but the enhancement in the total intensity is much less than two and decreases with increasing angle of incidence due to the fact that the double scattered intensity also decreases with incidence angle.

Figure 9 compares the total scattered intensity for s-s scattering from figures 7 and 8 with the results of 'exact' numerical calculations (based on the extinction theorem method for a perfect conductor [3]) and the experimental results of figure 3, for
Figure 7. Numerically calculated scattered intensities for a perfect conductor, using the double-scattering Kirchhoff approximation, for angles of incidence of $0^\circ$, $-30^\circ$ and $-60^\circ$ and s-s scattering (left) and p-p scattering (right). The wavelength is $0.63\ \mu m$. Each graph shows the doubly scattered intensity (lowest curve), single scattered (middle) and total intensity (coherent sum) (top curve).

$\lambda = 0.63\ \mu m$ and $1.15\ \mu m$ at three angles of incidence. The two numerical calculations agree well, showing that the Kirchhoff approximation is reasonable for these surface parameters (the average radius of curvature, defined as the inverse of the standard deviation of the surface curvature $2\sqrt{3\sigma/r^2}$, is $\approx 2.4\ \mu m$ for surface #46) and both agree well with the experimental measurements for zero angle of incidence. However, there is a clear discrepancy between experiment and numerical calculation for the $-30^\circ$ and $-60^\circ$ angles of incidence. (This general behaviour is also shown in the case of p-p scattering.)
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One possible cause of the discrepancy could be that the calculations are for a perfect conductor, whereas the experiments are for a real metal (gold). However, calculations by ourselves and others [5] show that, for these values of RMS surface height and correlation length, there is very little difference between the results for gold and for a perfect conductor, particularly for s-s scattering. One problem with most methods of calculation, including that used here, is that a very small length of surface is considered, giving rise to the possibility of an ‘end-effect’ error (e.g. due to long-range surface plasmons); however, the method of calculation of Saillard and Maystre [7] uses an extremely long length of surface with good agreement with the other calculations and poor agreement with the measurements at larger angles of incidence.

It seems, therefore, that there may be some aspect of the experiment that does not correspond to the calculations. Previous results for a Lambertian diffuser have
Figure 9. Comparison of Kirchhoff calculation (from figures 7 and 8), 'exact' numerical calculation and experiment (from figure 3), for s-s scattering at $\lambda = 0.63 \mu m$ and $1.15 \mu m$, and angles of incidence equal to $0^\circ$, $-30^\circ$ and $-60^\circ$. The solid curves are the Kirchhoff calculation, triangles the 'exact' calculation and crosses are the experimental results. Note the good agreement between the two numerical calculations but the departure of the experimental results for larger angles of incidence.

demonstrated that the scatterometer measures the correct quantity [10]. The measurement of surface properties might be in error. If one calculates the scattered intensity for, say, $-60^\circ$ angle of incidence for a surface that has an rms roughness 50% larger than the measured value, then reasonable agreement is obtained between experiment and numerical calculation. However, (a) it is extremely unlikely that such a gross error could occur (stylus tips effects are discussed by Church [12]) and (b) the agreement for $0^\circ$ angle of incidence is then very poor indeed, particularly as regards the location of the minima around the backscatter peak. Ishimaru and Chen [13] have shown that
a departure from Gaussianity of the correlation function could be responsible for the discrepancy, and the measured correlation does show a small departure from the Gaussian shape. However, it is notoriously difficult to estimate the correlation function of stylus traces and the departure shown in figure 2 is characteristic of inadequate de-trending of the mean; the method of manufacture of the surfaces strongly encourages a Gaussian correlation of surface height. The cause of this discrepancy for larger angles of incidence is therefore not resolved at the present time.

4. Summary

A set of scattering data for a one-dimensional surface at four wavelengths, three angles of incidence and two materials has been presented for critical comparison with numerical calculations and theoretical studies. A multiple scatter extension of the Kirchhoff approximation has been shown to provide additional physical evidence that the predominant cause of the enhanced backscatter peak is due to multiple scattering. There remains a significant disagreement between experiment and numerical calculations for large angles of incidence the cause of which is still unresolved.

The data presented in figures 3 to 6, together with sample Talystep traces, is available on a PC- or Macintosh-compatible diskette on application to the first author.

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