Measurements of the four-point coherence function by the use of the coherence enhancement phenomenon

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We present laboratory results for measurements of the four-point coherence function of a spherical wave in the region of coherence enhancement after backscattering through turbulence. Experimental results are compared with the theoretical predictions. We conclude that the shape of the four-point coherence function in our experiment depends on the value of the inner scale of turbulence. © 1996 Optical Society of America

1. Introduction

The phenomenon of coherence enhancement after double-passage propagation through turbulence has been experimentally observed for a basic system comprising a mirror target illuminated by two mutually coherent point sources. The degree of coherence enhancement obtained was increased significantly in an experiment that manipulated the polarization states of the illumination and reflected light. The physical nature of this phenomenon consists basically of the appearance of correlation in the backscattered field in regions surrounding the illumination sources and has been discussed in detail in previous publications. Essentially it is a result of reciprocity—any optical path through the turbulence that begins at one source and ends at the other has a corresponding partner that traverses the same path in the reverse direction. Experiments have demonstrated the existence of a region where coherence enhancement after backscattering through turbulence from an optically rough object may be observed. In Ref. 4 it was shown that the spatial distribution of the average intensity in the region of coherence enhancement is described by the four-point coherence function \( \Gamma_4 \) of a spherical wave. The four-point coherence function of the field is a subject of longstanding interest and has a considerable literature. Experimental data on the behavior of \( \Gamma_4 \) are, however, not so common. Here we present the results of quantitative measurements of \( \Gamma_4 \), using an interferometric system designed for the observation of the coherence enhancement phenomenon.

2. Experimental System and Rationale

The experimental system is depicted schematically in Fig. 1. A point source from a linearly polarized He–Ne laser focuses on the surface of a birefringent crystal. The birefringent crystal produces two orthogonally polarized waves having ordinary (\( \sigma \)) and extraordinary (\( \varepsilon \)) polarizations. The crystal was oriented so as to give \( \sigma \) and \( \varepsilon \) beams of equal intensity, and the lateral spatial shift \( d \) between the two wave fronts at the exit surface of the crystal was equal to 2.5 mm. Behind the crystal, a quarter-wave plate was installed with its fast axis oriented such that linearly polarized radiation passing through it has its polarization state transformed and becomes circularly polarized. This is achieved when the fast axis is set at an angle of +45° and −45° for the \( \sigma \) and \( \varepsilon \) rays, respectively. After passage through the quarter-wave plate and turbulent cell, the light scattered from the incoherently reflecting target (a rotating rough surface, Scotch-lit) back through the cell and optical system. The coherence radius of a spherical wave passing through the turbulence in the plane of the surface was much larger than the characteristic transverse scale of the target roughness. We can also consider our scattering surface to
be optically rough. Distance $L$ between the point source and target was equal to 2 m. A line-scan array detector, controlled by a personal computer, was used to record the radiation at the source plane.

After double passage through the quarter-wave plate, the initially ordinary polarized wave is transformed into an extraordinary polarized one, and the initially extraordinary polarized wave is transformed into an ordinary polarized one. On each passage the birefringent beam splitter has no effect on the ordinary polarized wave but introduces a spatial shift $d$ in the wave front of the extraordinary polarized wave. Hence the reflected field in the plane of the illumination source (i.e., after the second pass through the birefringent crystal) is the sum of the orthogonally polarized waves. With the help of a suitably oriented polarizer, we may observe the interference of these two waves.

Let us now consider quantitatively the formation of the interference pattern observed through the polarizer. Let a point source be located in the plane $z = 0$ (on the surface of the birefringent crystal) at the point $r_s$, and let an illuminated target characterized by a complex amplitude reflection coefficient, $O(r)$, be located in the plane $z = L$. We denote the scalar field in the plane $z = L$ at point $r_t$ because of a spherical wave radiated by a point source located in the plane $z = 0$ at $r_1$ as $G(0, r_1 \rightarrow L, r_t)$. In the general case, one can represent the field $G(0, r \rightarrow L, p)$ in the following way:

$$G(0, r \rightarrow L, p) = G(0, r \rightarrow L, p)V(0, r \rightarrow L, p),$$  

where

$$g(0, r \rightarrow L, p) = \frac{k}{2 \pi i L} \exp \left[ \frac{ik}{2L} (r - p)^2 \right].$$

is the Fresnel approximation for a spherical wave field in a homogeneous medium, $k = 2\pi/\lambda$, $\lambda$ is the wavelength, and $V$ is the perturbation of this wave introduced by the inhomogeneities of the medium.

Let us consider separately the propagation of ordinary and extraordinary polarized waves through our interferometric system. The birefringent crystal has no effect on the initially ordinary polarized wave, and consequently amplitude $\mathcal{G}_1$ of the corresponding circularly polarized wave (after passage through the quarter-wave plate) on the target sur-
vided it does not change the polarization state of the illuminating wave.

In the presence of turbulence, the expression for the average distribution of intensity $I(r)$ in the source plane is more complicated. However, when the scale of target roughness is much smaller than the coherence radius of the incident spherical wave, then one can assume that

$$O(p_+ + p/2)O^*(p_+ + p/2) \sim |O(p_+)|^2 \delta(p).$$

(10)

In this case the average distribution of intensity $|I(r)|^2$ is described by the following expression:

$$|I(r)|^2 = \text{const} \left( I_{bs}(r - r_s + d) + I_{bs}(r - r_s - d) + 2 \Re \left[ \Gamma_4(r - r_s, d) \exp \left[ i \frac{k}{L} (r - r_s, d) \right] \right] \right)$$

(11)

where $\Gamma_4$ is the four-point coherence function of perturbation $V$ on the spherical wave

$$\Gamma_4(r, d) = \langle V(0, \rho - L, p_+)V^*(0, \rho + r - L, p_+) \rangle \times \langle V(0, \rho + d + L - r_s, p_+) \rangle \times \langle 0, \rho + d - L, p_+ \rangle,$$

(12)

and $I_{bs}(r) = \Gamma_4(r, 0)$ is the average distribution of intensity of backscattered light in the absence of the crystal and quarter-wave plate. Here $I_{bs}(r - r_s + d)$ and $I_{bs}(r - r_s - d)$ describe the backscattering enhancement peaks when $r = r_s + d$ and $r = r_s - d$. The third term in Eq. (12) describes the peak of coherence enhancement in the region near the illumination source location $r = r_s$.

3. Experimental Measurements

The measurements were made for weak intensity fluctuations produced by the turbulent cell. The normalized variance of the intensity fluctuations, $\beta^2$, was approximately 0.3, and its value was measured both directly with the diode array installed at the target plane and illumination by a point source through the turbulence and also by measurements of the enhanced backscattering coefficient. Both of these measurements were in good agreement.

The intensity distribution measured at the source plane in the absence of turbulence is shown in Fig. 2. It should be noted that with the object stationary we observe this interference pattern with a speckle structure superimposed. A typical time-averaged intensity distribution measured in the presence of turbulence with a rotating target is shown in Fig. 3. This distribution was obtained for strong intensity fluctuations when the normalized variance of intensity fluctuations was approximately unity, thus increasing the visibility of the backscattering enhancement peaks. These two figures are in good qualitative agreement with Eqs. (2) and (3).

Function $\Gamma_4$ was measured when the elements of the diode array were aligned along the central interferometric fringe (see Fig. 3). During these measurements the diode array was covered by a narrow slit of width $\sim 0.1 \text{ mm}$. The magnification of the lens used as the objective was equal to 2, and consequently the width of the interferometric fringes in the diode array plane was $\sim 1 \text{ mm}$; that is, the slit width was smaller than the width of the interferometric fringes. It follows from Eq. (2) that in this case, if $r$ is orthogonal to $d$, the observed intensity distribution is described by

$$\langle I(r) \rangle = \text{const} + \Gamma_4(r - r_s, d).$$

(13)

The measured shape of $\Gamma_4(r, d)$ is shown in Fig. 4 (curve 1) as a function of the variable $r$, together with the results of theoretical calculations. Because the length of our turbulent cell was significantly smaller than the distance between the target and the illumination source, the calculations can be carried out...
with the thin-phase screen model. In this model the expression for field $V$ of a spherical wave is given by

$$V(0,\rho \rightarrow L, p) = \frac{k}{2i\pi \varepsilon_{ef}} \int d^2q \exp\left[\frac{i}{2\varepsilon_{ef}} q_2\right]$$

$$+ \frac{ik}{2} v\left(q + \frac{L - z}{L} \rho + \frac{z}{L} p\right),$$

(14)

where $\frac{ik}{2} v(p)$ is the phase fluctuation caused by the screen and $z$ is the distance between the plane of the phase screen and the plane of observation, $z_{ef} = zL/(L - z)$ (in our case the turbulent cell was located in the middle of the path and $z_{ef} = L/4$). The expression for function $\Gamma_4$ is

$$\Gamma_4(r, p) = \left(\frac{k}{2\pi \varepsilon_{ef}}\right)^2 \int d^2q_1 d^2q_2 \exp\left[\frac{i}{z_{ef}} q_1 q_2\right]$$

$$- \Psi\left(q_1 + \frac{L - z}{L} r, q_2 + \frac{L - z}{L} p\right),$$

(15)

where

$$\Psi(p_1, p_2) = 2D(p_1) + 2D(p_2) - D(p_1 + p_2) - D(p_1 - p_2),$$

(16)

and $D$ is the structure function of the phase screen. Equation (15) was calculated analytically by the use of a Taylor expansion for $\Psi$ up to second-order terms in variables $p_1$ and $p_2$.

As shown in Ref. 8, in calculations of the statistical properties of waves passing through our turbulent cell, it is essential to take into account the inner scale $l_0$. For our calculation of function $D$, the following expression for the power spectrum was used:

$$\Phi(\kappa) = \frac{A}{\kappa^{1/3} \phi(\kappa)},$$

(17)

where function $\phi(\kappa)$ describes the decay of the spectrum caused by dissipation at large wave numbers $\kappa$. Two expressions for function $\phi(\kappa)$ were tested. The first one was suggested by Tatarskii:

$$\phi(\kappa) = \exp\left[-\frac{\kappa L_0}{5.92}\right].$$

(18)

The second one was calculated according to the Hill model with the Prandtl number for water equal to 7.

The constant, $A$, was determined by the use of an independent measurement of the coherence radius of a spherical wave. This was obtained by analysis of the time-averaged image of a point source obscured by turbulence. In our experiment the value of the coherence radius was approximately equal to 0.64 mm. In Fig. 4, curve 2 was calculated for a pure Kolmogorov spectrum $\phi(\kappa) = 1$. By varying $\kappa_0$, we may obtain a family of theoretical curves for $\Gamma_4$.

Curve 3 shows the best least-squares fit between our experimental data and the theoretical calculations for both models of the spectrum (the curves for Tatarskii and the Hill model coincide within the graphical accuracy). For the Tatarskii spectrum, this corresponds to a value of $\kappa_0 = 2.5$. For the Hill spectrum the best correspondence was achieved at a Kolmogorov scale equal to 1.2 mm. The corresponding value of the inner scale was estimated as $l_0 = 1.6$ mm. The value of the normalized variance of intensity fluctuations $\beta^2$ at the target plane calculated with the Hill spectrum $\beta^2 = \exp[\beta_0^2] - 1 = 0.3$, where $\beta_0^2$ is the calculated value of the log intensity variance, coincides with the experimentally measured value $\beta^2 = 0.3$; for the Tatarskii spectrum there is a significant difference between these two values $\beta^2 = \exp[\beta_0^2] - 1 = 0.45$.

A comparison of curves 2 and 3 in Fig. 4 shows that the shape of the four-point coherence function depends on the value of the inner scale of the turbulence. A similar dependence of the second moment of intensity on the inner scale value is widely used for inner scale measurements by scintillation techniques.

4. Conclusions

It is well known that under backscattering of point source radiation from a rough surface through turbulence, the spatial distribution of average intensity is described by the intensity correlation function of the spherical wave. Thus the simple backscattering configuration allows us to determine only a particular case of the four-point coherence function by observing the enhanced backscattering peak.

Using a special interferometric system, we have
illuminated a rough surface through turbulence with a point source and have observed the interference pattern in the source plane. We have shown that the time-averaged intensity distribution in this interferometric pattern is described by the four-point coherence function of a spherical wave propagated through turbulence. Measurements of this function were conducted and showed good agreement between experimental data and results of calculations.

The major advantage of the proposed method in comparison with the existing direct measurement technique is the simplicity of the measurement performance and the availability of this function shape determination through one measurement. This study revealed a new possible application of backscattering for the investigation of the statistical properties of waves propagating through turbulence.

This research was carried out with partial support from the Royal Society (London) and the International Science Foundation (grant N MH7000) and from the UK Engineering and Physical Sciences Research Council (grant GR/H 33169).

References