Astronomical adaptive optics in parameter space

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We present a general, undiscretized formulation of astronomical adaptive optics that encompasses arbitrary guide star sources and deformable mirror configurations. It is shown that wave-front measurements can be represented as samples of an integral transform of the turbulence perturbation and also that the desired information for adaptive correction is a subset of this transformed space. Some properties of this space are explored, and their implications for adaptive optics are discussed. © 2000 Optical Society of America

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With their ability to correct for the image degrading effects of atmospheric turbulence, adaptive optics systems are becoming common companions to astronomical telescopes. Because of limitations in sky coverage and field of view, however, the classic arrangement, in which a natural reference star is used to control a single correcting mirror, is losing popularity in favor of methods that use laser guide stars and (or) multiconjugate correction.[1] But such techniques inevitably bring additional degrees of freedom that need optimization and additional information that requires processing, and in this Letter we describe the astronomical adaptive optics problem in a general manner that, we believe, facilitates the accomplishment of such tasks.

For most problems in astronomical adaptive optics it is sufficient to assume that the atmosphere imposes only weak phase perturbations on light propagating through it, thereby allowing the phase at any point in the telescope pupil to be approximated by the integrated perturbation along a straight ray path from the source.[2] Thus, if we consider an arbitrary phase perturbing volume $\psi(\mathbf{r})$ and a point source above the turbulence (representing a laser guide star) at the Cartesian position (ξ_x, ξ_y, H), then the phase at position $(\eta_x, \eta_y, 0)$ in the pupil plane of the telescope is given by

$$\Psi(\boldsymbol{\eta}, \boldsymbol{\xi}) = \left(1 + \left|\frac{\boldsymbol{\eta} - \boldsymbol{\xi}}{H}\right|^2\right)^{1/2} \times \int \psi(\mathbf{r})\delta^2\left(x - \boldsymbol{\eta} + \frac{\boldsymbol{\eta} - \boldsymbol{\xi}}{H} z\right)\mathrm{d}\mathbf{r}, \quad (1)$$

where $\mathbf{x} = (x, y)$, $\delta^2(\mathbf{x}) = \delta(x)\delta(y)$, and $\boldsymbol{\eta}$ and $\boldsymbol{\xi}$ are horizontal vectors in planes $z = 0$ and $z = H$, respectively (see Fig. 1). If, as is often assumed, the turbulence can be approximated by several thin horizontal layers $\psi_j(\mathbf{x})$ at respective heights h_j, that is, if

$$\psi(\mathbf{r}) = \sum_j \psi_j(\mathbf{x})\delta(z - h_j), \quad (2)$$

Eq. (1) will yield the familiar expression[3]

$$\Psi(\boldsymbol{\eta}, \boldsymbol{\xi}) = \sum_j \psi_j(\tilde{h}_j\boldsymbol{\xi} + (1 - \tilde{h}_j)\boldsymbol{\eta}), \quad (3)$$

where $\tilde{h}_j = h_j/H$ and we have neglected the rooted factor in Eq. (1) because the height of the guide star, H, is typically much larger than the magnitudes of $\boldsymbol{\eta}$ and $\boldsymbol{\xi}$. We ignore, for the moment, the loss of tilt information inherent in laser guide star measurements as well as the inability to measure the piston of a sensed wave front.

Although $\Psi(\boldsymbol{\eta}, \boldsymbol{\xi})$ is derived to describe the wave front from a given laser guide star at $\boldsymbol{\xi}$, we point out that it describes, somewhat more generally, the phase measurement obtained from any ray. We can consider, for instance, a ray emanating from a natural star in the direction of a unit vector $\hat{a}$. It is a simple exercise to show that, to terminate at $\boldsymbol{\eta}$ in the pupil plane, the ray must pass through the $z = H$ plane at $\boldsymbol{\xi} = (\hat{a}/\hat{a} \cdot \hat{z} - \hat{z})H + \boldsymbol{\eta}$. Clearly it is possible to describe any wave-front measurement, $w(\boldsymbol{\eta})$, say, as a sample of the total phase measurement space thus:

$$w(\boldsymbol{\eta}) = \Psi(\boldsymbol{\eta}, \boldsymbol{\xi}_s(\boldsymbol{\eta})), \quad (4)$$

where $\boldsymbol{\eta}$ is confined to the telescope pupil and $\boldsymbol{\xi}_s(\boldsymbol{\eta})$ is a function that depends on the source type, as summarized in Table 1. This sampling can also be visualized in a plane if the turbulence is two dimensional, as is shown in Fig. 2. We emphasize that this space need

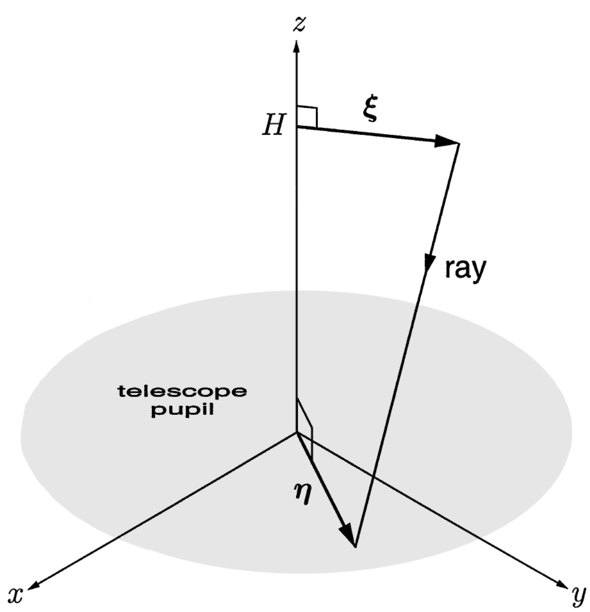

Fig. 1. Straight ray of light integrates the phase perturbation along its path. This path can be parameterized by $\boldsymbol{\eta}$ and $\boldsymbol{\xi}$, its intersections with the two horizontal planes shown.

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Table 1. Sampling Functions for Various Sources

Source	Sampling function $\boldsymbol{\xi}_s(\boldsymbol{\eta})$
Laser guide star at $\boldsymbol{\xi}_0$	$\boldsymbol{\xi}_0$
Natural guide star at $\hat{a}$	$(\hat{a}/\hat{a}\cdot\hat{z} - \hat{z})H + \boldsymbol{\eta}$
Star at zenith	$\boldsymbol{\eta}$

not be parameterized by $\boldsymbol{\eta}$ and $\boldsymbol{\xi}$, and one can choose any set of parameters that is sufficient to describe a ray.

With this generalization, one can view measurement space Ψ to be simply an integral transform of the turbulence perturbation ψ as defined in Eq. (1), and it is worth noting some of the properties of this transformation. First, the transformed space is complete, which is to say that knowledge of all the space is sufficient to enable one to reconstruct fully, using an appropriate inversion, the perturbation volume from which it arose. To show this, we first form an intermediate space of only three parameters by integrating Ψ along all pairs of parallel lines in the $\boldsymbol{\eta}$ and $\boldsymbol{\xi}$ planes. This is tantamount to finding the turbulence perturbation integrated along each possible plane (rather than ray) through the turbulence, and the resultant space is the three-dimensional Radon space. The Radon transform has been studied extensively and is a common tool for tomography, an inverse problem in which one tries to infer the internal structure of a volume from measurements of radiation passing through it.[4] Subject to the perturbing volume's being finite, the inverse Radon transform can completely reconstruct it from full knowledge of the Radon space. The fact that one parameter of Ψ can be integrated away to still form a complete space highlights another interesting property of the measurement space, that it contains redundancy. This suggests that it may be possible to choose the sampling of the space to maximize information retrieval, but such an analysis is beyond the scope of this Letter. Incidentally, no redundancy occurs in the two-dimensional case because Ψ is already the two-dimensional Radon transform of ψ.

Whereas knowing how to probe the measurement space is important, we must also consider what information is desired from that space. In many analyses there is assumed to be a stellar target object (which, for the sake of simplicity, we assume to be at zenith) whose distorted wave front we wish to correct with a deformable mirror.[5] As we have already shown, this wave front is just $\Psi(\boldsymbol{\eta}, \boldsymbol{\eta})$ for $\boldsymbol{\eta}$ within the pupil, so the problem is clearly one of interpolation between and (or) extrapolation beyond the various samples of Ψ measured by reference stars. It is evident from Fig. 2 that there is no hope of filling in the unsampled space without some *a priori* knowledge about the turbulent volume. Such knowledge can range from knowing only that the turbulence obeys Kolmogorov statistics with a measurable Fried parameter to knowing the precise heights and turbulence strengths of the various layers in the turbulent region. These assumptions and measured quantities of the turbulence are also vital in estimating the piston and, in the case of laser guide stars, the tilt that the measurement samples can lack.

A somewhat more difficult challenge is to try to apply good correction over a finite field of view, and in this case the precise role of the deformable mirror needs be taken into account. The effect of a deformable mirror is the same as that of placing a known phase perturbation layer into the atmosphere at the height conjugate to the mirror position. So, for a mirror shape $m(\mathbf{x})$ at conjugate height h_c, we add $m(\mathbf{x})\delta(z-h)$ to $\psi(\mathbf{r})$ or, equivalently, add

$$M(\boldsymbol{\eta}, \boldsymbol{\xi}) = m(\tilde{h}\boldsymbol{\xi} + (1-\tilde{h})\boldsymbol{\eta}) \tag{5}$$

to $\Psi(\boldsymbol{\eta}, \boldsymbol{\xi})$ [as in Eq. (3)], to produce the resultant phase residual. In the two-dimensional case a layer can be depicted in the parameter space, as shown in Fig. 3. Clearly, even if we assume that any deformation is possible, a single mirror is limited in its ability to match the aberrations of an arbitrary turbulence volume over a range of directions, and this is the motivation for using multiple deformable mirrors at different conjugate heights. With a given number of mirrors, the problem can now be seen as one of fitting the sum of a set of functions $M_i(\boldsymbol{\eta}, \boldsymbol{\xi})$ (each representing the effect of one mirror) of the form described above to a particular region, either known or estimated, of $\Psi(\boldsymbol{\eta}, \boldsymbol{\xi})$. There are several criteria that can be used to make an optimal fitting, but as yet there is no consensus on which criteria are the most useful for the astronomer. The simplest criterion to apply is to make an optimal fit to those samples of Ψ measured from the various guide stars, that is, to minimize, with respect to the mirror forms and conjugate heights, the quantity

$$\sigma^2 = \sum_k \int_P \mathrm{d}\boldsymbol{\eta} \left| \Psi(\boldsymbol{\eta}, \boldsymbol{\xi}_k(\boldsymbol{\eta})) + \sum_i M_i(\boldsymbol{\eta}, \boldsymbol{\xi}_k(\boldsymbol{\eta})) \right|^2, \tag{6}$$

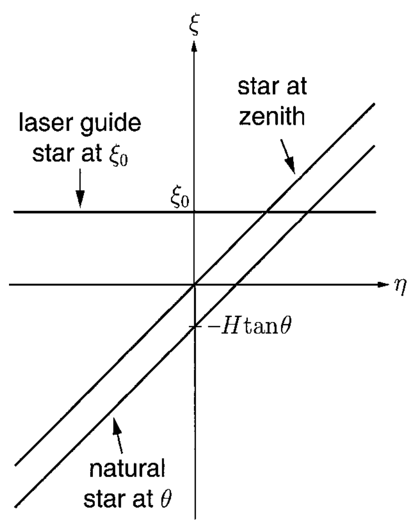

Fig. 2. When turbulence and light propagation are confined to a vertical plane the measurement space can be parameterized by scalar quantities η and ξ. Thicker lines show how this space is sampled by a laser guide star at $\xi = \xi_0$, a natural guide star with (anticlockwise) zenith distance θ, and a natural star at zenith.

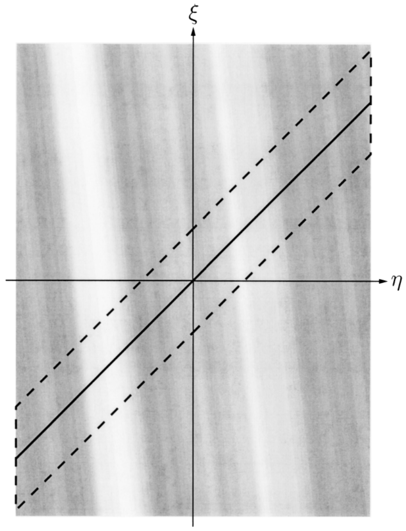

Fig. 3. Gray-scale representation of a deformable mirror function in $\eta\xi$ parameter space for the two-dimensional problem. The mirror function $m(x)$ is mapped directly onto line $\xi = \eta$, and lines of equiphase have slope $(1 - H/h)$, where h is the conjugation height of the mirror ($h/H = 0.1$ in the depicted case). The area inside the dashed lines indicates a region over which functions such as this must match $\Psi(\eta, \xi)$ to correct for a finite field view.

where $\boldsymbol{\xi}_k(\boldsymbol{\eta})$ represents the sampling function for the kth guide star as given in Table 1 and P represents the telescope pupil. One could also contrive to optimize the isoplanatic patch size about a given direction or to maintain as constant a residual error over a given field of view. Implementing these criteria requires that one have the *a priori* information mentioned above to estimate the unsampled portions of Ψ that are required for optimization. Performing such optimizations is beyond the scope of this Letter and, in any case, is the subject of some previous papers.[6,7] Here we wish only to stress that the formulation above is not restricted to particular guide star constellations, deformable mirror characteristics, or spatial decompositions of the turbulence and therefore that it can be used to optimize these as well.

In summary, we have shown that the measurable information and the desired information in astronomical adaptive optics are subsets of a space that can be defined by an integral transform of the turbulence perturbation. Consequently the problem cannot be described as a tomographic one, despite the relationship that this space has to Radon space, but is rather a problem of interpolation and (or) fitting. This is not to suggest that the many so-called tomographic methods that have been proposed for tackling the problem are misconceived but only that the terminology is used somewhat liberally to describe them. We have also described an astronomical adaptive optics system without recourse to discretization or decomposition of any kind, and we believe that such a formulation can aid in optimization of the parameters hitherto considered fixed.

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References

1. R. Raggazoni, E. Marchetti, and G. Valente, Nature **403,** 54 (2000).
2. F. Roddier, ed., *Adaptive Optics in Astronomy* (Cambridge U. Press, Cambridge, 1999), Chap. 2.
3. D. C. Johnston and B. M. Walsh, J. Opt. Soc. Am. A **11,** 394 (1994).
4. S. R. Deans, *The Radon Transform and Some of Its Applications* (Wiley, New York, 1983).
5. R. Raggazoni, E. Marchetti, and F. Rigaut, Astron. Astrophys. **342,** L53 (1999).
6. T. Fusco, J.-M. Conan, V. Michau, L. M. Mugnier, and G. Rousset, Proc. SPIE **3763,** 125 (1999).
7. B. L. Ellerbroek and F. J. Rigaut, in *Adaptive Optical Systems Technology*, P. L. Wizinowich, ed., Proc. SPIE **4007** (to be published).