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Letter

Adaptive Optics as a Spatial Coherence Modifier

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The possibility of adaptive optics as a spatial coherence modifier is discussed. The initial field to be corrected is produced in such a way that a quasi-monochromatic plane wave of light is incident on a moving diffuser plate, so that it is spatially partially coherent. It is shown that adaptive optics serves to enhance the spatial coherence of the resultant field and that the magnitude of the enhancement depends on the spatial coherence of the initial field to be corrected. The results are illustrated by numerical examples based on an idealized low-order adaptive optics system.

Key words: adaptive optics, spatial coherence, Zernike modes, low-order adaptive optics, partial wave-front correction

In recent years a good deal of attention has been devoted to adaptive optics as a powerful technique to achieve a real-time restoration of a blurred astronomical image.[1] Here the basic functions of adaptive optics are to measure the wave-front deformation that is due to atmospheric turbulence and to drive a wave-front corrector to compensate for the deformation in real time. Therefore, adaptive optics systems capable of full correction of the deformed wave-front provide nearly diffraction-limited imaging, though they are rather complicated and expensive. On the other hand, it has been shown that simpler adaptive optics systems for correcting only low-order aberrations can significantly improve astronomical images.[2] Owing to its simple and inexpensive nature, such low-order adaptive optics for partial wave-front correction has become the subject of growing interest[3] and found useful applications beyond astronomy.[4,5]

In general, the main objective of adaptive optics is to improve the image quality degraded by fluctuating phase disturbance such as atmospheric turbulence. The performance of an (incoherent) imaging system is characterized by its spatial frequency response, i.e., the optical transfer function (OTF). The OTF of the overall system is given by the product of the OTF of the system without the random phase disturbance and the spatial correlation function of the field after passing through the phase disturbance, assuming that the random field is statistically stationary.[6] In other words, the OTF due to the random part of the field is equivalent to the spatial coherence of the field. We can therefore view adaptive optics as bringing about changes in the OTF of the overall system by modifying the spatial coherence of the field.

In this Letter, we deal theoretically with the effect of adaptive optics on the spatial coherence of the field to explore the interpretation of adaptive optics as a spatial coherence modifier. It is to be noted that studies on controlling the spatial coherence of the field have been of practical interest in connection with photolithography in the semiconductor industry and other applications,[7] since the spatial coherence of the field plays an important role in the performance of imaging systems.

Figure 1 shows a schematic diagram for analysis. An initial field to be corrected is assumed to be produced in such a way that a quasi-monochromatic plane wave of light is incident on a moving diffuser plate D, which is assumed to be purely a phase object, through a circular aperture A located just in front of the diffuser plate, so that it is spatially partially coherent. In this context the "moving" of the diffuser plate contributes simply to the average of the field after passing through the diffuser plate; one can envisage other means of modifying the spatial coherence such as acousto-optic modulation.[8] Due to the two sets of relay lenses, both a wave-front corrector and an output plane are conjugate with the diffuser plate. By the arrangement for adaptive optics depicted in Fig. 1, some of the low-order Zernike modes (i.e., the low-order aberrations specified by Zernike polynomials) in the wave-front deformation are assumed to be completely corrected.

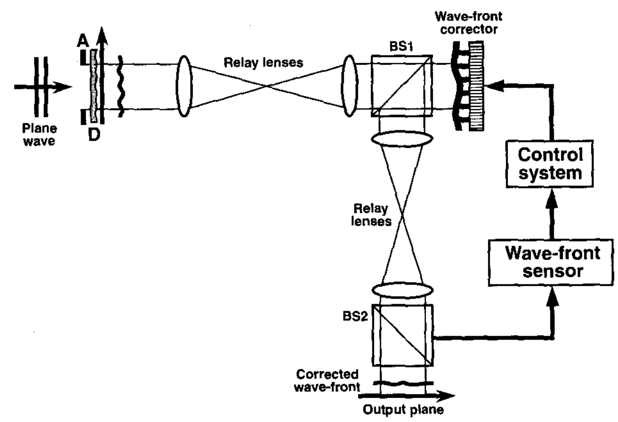

Fig. 1. Schematic diagram for analysis: A, circular aperture; D, moving diffuser plate; BS1-2, beam splitters.

Let $\phi(R\boldsymbol{\rho}, t)$ be a (random) phase of the diffuser plate where R is the radius of the circular aperture and $\boldsymbol{\rho}$ is the normalized position vector in the diffuser plane. The field just behind the diffuser plate is thus given by $A(t)\exp[i\phi(R\boldsymbol{\rho}, t)]\exp(-2\pi i\bar{\nu}t)$, where $A(t)$ is the amplitude of the incident quasi-monochromatic plane wave of light and $\bar{\nu}$ is its mean frequency. In the analysis to follow, we assume that the random fluctuations of the phase obey Gaussian statistics with a zero mean $\langle\phi(\boldsymbol{\rho}, t)\rangle = 0$ and that its second-order spatial correlation function is given by the expression

$$\langle\phi(\boldsymbol{\rho}_1,t)\phi(\boldsymbol{\rho}_2,t)\rangle=\phi_0^2\exp\left(-\frac{|\boldsymbol{\rho}_1-\boldsymbol{\rho}_2|^2}{2\sigma_\phi^2}\right),\tag{1}$$

where $\phi_0=\sqrt{\langle|\phi(\boldsymbol{\rho},t)|^2\rangle}$, σ_ϕ is a positive constant characterizing the correlation width of the diffuser phase, and the angular brackets denote a time average of the diffuser phase.

To analyze the spatial coherence of the field after the Zernike-mode correction, we first decompose the initial wave-front (phase) into Zernike polynomials $Z_j(\boldsymbol{\rho})$:

$$\phi(R\boldsymbol{\rho})=\sum_j b_j Z_j(\boldsymbol{\rho}),\tag{2}$$

where b_j are the decomposition coefficients given by

$$b_j=\iint\phi(R\boldsymbol{\rho})w(\boldsymbol{\rho})Z_j(\boldsymbol{\rho})\mathrm{d}^2\rho\tag{3}$$

and the weighting function $w(\boldsymbol{\rho})$ is given by $1/\pi$ for $|\boldsymbol{\rho}|\le 1$ and 0 for $|\boldsymbol{\rho}|>1$. Here and hereafter, the explicit dependence on t in $\phi(R\boldsymbol{\rho},t)$ is omitted for brevity, and the integration extends over the whole space. Moreover, the Zernike polynomials are defined using polar coordinates $\boldsymbol{\rho}=(\rho\cos\theta,\rho\sin\theta)$ by

$$Z_j(\rho,\theta)=\sqrt{n+1}\times\begin{cases}R_n^m(\rho)\sqrt{2}\cos m\theta, & j\text{ even},\ m\neq 0\\ R_n^m(\rho)\sqrt{2}\sin m\theta, & j\text{ odd},\ m\neq 0\\ R_n^0(\rho), & m=0,\end{cases}\tag{4}$$

where

$$R_n^m(\rho)=\sum_{s=0}^{(n-m)/2}\frac{(-1)^s(n-s)!}{s![(n+m)/2-s]![(n-m)/2-s]!}\rho^{n-2s}.\tag{5}$$

The index j indicating an ordering of the polynomials (i.e., the mode number), which is a function of both the radial degree n and the azimuthal frequency m, is given in the same manner as Noll.[9]

Suppose now that some of the low-order Zernike modes specified by $j=l_1,l_2,\cdots$ are completely removed from the initial field so that the resultant wave-front becomes

$$\hat{\phi}(R\boldsymbol{\rho})=\sum\nolimits_{j\neq l_1,l_2,\cdots}b_jZ_j(\boldsymbol{\rho}).\tag{6}$$

Then the instantaneous field after correction is represented, using Eq. (6), by

$$V_C(R\boldsymbol{\rho},t)=A(t)\exp[i\hat{\phi}(R\boldsymbol{\rho})]\exp(-2\pi i\bar{\nu}t).\tag{7}$$

To examine the spatial coherence properties of the resultant (quasi-monochromatic) field, we introduce the mutual intensity of the field defined by[10]

$$J_C(R\boldsymbol{\rho}_1,R\boldsymbol{\rho}_2)=\langle V_C^*(R\boldsymbol{\rho}_1,t)V_C(R\boldsymbol{\rho}_2,t)\rangle,\tag{8}$$

and its normalized quantity, which is referred to as the complex degree of coherence, defined by

$$\gamma_C(R\boldsymbol{\rho}_1,R\boldsymbol{\rho}_2)=\frac{J_C(R\boldsymbol{\rho}_1,R\boldsymbol{\rho}_2)}{\sqrt{J_C(R\boldsymbol{\rho}_1,R\boldsymbol{\rho}_1)}\sqrt{J_C(R\boldsymbol{\rho}_2,R\boldsymbol{\rho}_2)}},\tag{9}$$

where the asterisk denotes the complex conjugate. To quantify the spatial coherence, we employ the degree of coherence $|\gamma_C(R\boldsymbol{\rho}_1,R\boldsymbol{\rho}_2)|$ (which is defined as the absolute value of Eq. (9)), as usual, in the following analysis. On substituting from Eqs. (7) and (8) into Eq. (9), we have

$$|\gamma_C(R\boldsymbol{\rho}_1,R\boldsymbol{\rho}_2)|=|\langle\exp[i\{\hat{\phi}(R\boldsymbol{\rho}_2)-\hat{\phi}(R\boldsymbol{\rho}_1)\}]\rangle|.\tag{10}$$

By taking into account that the resultant wave-front $\hat{\phi}(R\boldsymbol{\rho})$ also obeys Gaussian statistics with a zero mean,[11] we can readily show that Eq. (10) becomes[12]

$$\begin{aligned}|\gamma_C(R\boldsymbol{\rho}_1,R\boldsymbol{\rho}_2)|&=\exp\left[-\frac{1}{2}\langle\{\hat{\phi}(R\boldsymbol{\rho}_2)-\hat{\phi}(R\boldsymbol{\rho}_1)\}^2\rangle\right]\\&=\exp\left[-\frac{1}{2}\{\langle|\hat{\phi}(R\boldsymbol{\rho}_1)|^2\rangle+\langle|\hat{\phi}(R\boldsymbol{\rho}_2)|^2\rangle\}\right.\\&\qquad\left.+\langle\hat{\phi}^*(R\boldsymbol{\rho}_1)\hat{\phi}(R\boldsymbol{\rho}_2)\rangle\right],\end{aligned}\tag{11}$$

where

$$\begin{aligned}\langle\hat{\phi}^*(R\boldsymbol{\rho}_1)\hat{\phi}(R\boldsymbol{\rho}_2)\rangle&=\langle\hat{\phi}^*(R\boldsymbol{\rho}_2)\hat{\phi}(R\boldsymbol{\rho}_1)\rangle\\&=\sum\nolimits_{j\neq l_1,l_2,\cdots}\sum\nolimits_{j'\neq l_1,l_2,\cdots}\langle b_j^*b_{j'}\rangle Z_j^*(\boldsymbol{\rho}_1)Z_{j'}(\boldsymbol{\rho}_2)\end{aligned}\tag{12}$$

and

$$\langle|\hat{\phi}(R\boldsymbol{\rho})|^2\rangle=\sum\nolimits_{j\neq l_1,l_2,\cdots}\sum\nolimits_{j'\neq l_1,l_2,\cdots}\langle b_j^*b_{j'}\rangle Z_j^*(\boldsymbol{\rho})Z_{j'}(\boldsymbol{\rho}).\tag{13}$$

It is obvious from Eq. (11), together with Eqs. (12) and (13), that the degree of coherence of the resultant field is determined if all the correlations $\langle b_j^*b_{j'}\rangle$ of the decomposition coefficients are given beforehand. Values of $\langle b_j^*b_{j'}\rangle$ are obtained by the expression

$$\begin{aligned}\langle b_j^*b_{j'}\rangle=&\iint\iint\langle\phi^*(R\boldsymbol{\rho})\phi(R\boldsymbol{\rho}')\rangle\\&\times w^*(\boldsymbol{\rho})Z_j^*(\boldsymbol{\rho})w(\boldsymbol{\rho}')Z_{j'}(\boldsymbol{\rho}')\mathrm{d}^2\rho\,\mathrm{d}^2\rho',\end{aligned}\tag{14}$$

which is derived using Eq. (3). Equation (14) may be readily evaluated in Fourier space, in accordance with Noll[9] and others.[13] After some lengthy calculations, we have

$$\begin{aligned}\langle b_j^*b_{j'}\rangle=&\phi_0^2\left(\frac{R}{\sigma_\phi}\right)^{n+n'}2^{-\frac{n+n'}{2}}(-1)^{\frac{n+n'}{2}-m}\\&\times\sqrt{(n+1)(n'+1)}\delta_{mm'}\frac{\Gamma[(n+n')/2+1]}{\Gamma(n+2)\Gamma(n'+2)}\\&\times{}_3F_3\left[\frac{n+n'+3}{2},\frac{n+n'}{2}+2,\frac{n+n'}{2}+1;\right.\\&\qquad\left.n+2,n'+2,n+n'+3;\ -2\left(\frac{R}{\sigma_\phi}\right)^2\right]\end{aligned}\tag{15}$$

for $j-j'$: even, and $\langle b_j^*b_{j'}\rangle=0$ for $j-j'$: odd, where Γ and ${}_3F_3$ are the Gamma function and the generalized hypergeometric function, respectively, and $\delta_{mm'}$ denotes the Kronecker delta function. Once the ratio σ_ϕ/R and the value ϕ_0 are given, one can calculate all the values of $\langle b_j^*b_{j'}\rangle$.

To show some numerical examples, we evaluate the degree of coherence of the field at two points specified by the position vectors $\boldsymbol{\rho}_1=\boldsymbol{\rho}_+\equiv(\frac{1}{2}\rho,0)$ and $\boldsymbol{\rho}_2=\boldsymbol{\rho}_-\equiv(-\frac{1}{2}\rho,0)$.

Let us first consider that the wave-front correction is suspended. On substituting from Eq. (1) into Eq. (11), we

obtain for the degree of coherence of the uncorrected field (i.e., the field just behind the diffuser plate) the expression

$$|\gamma_C(R\rho_+, R\rho_-)| \equiv |\gamma_C(\rho)| = \exp\left[-\phi_0^2\left\{1 - \exp\left[-\frac{\rho^2}{2(\sigma_\phi/R)^2}\right]\right\}\right]. \quad (16)$$

Consider the case of low-order Zernike-mode correction. In the present analysis, we assume that our system completely corrects the following 9 Zernike modes: $j = 1$ (piston), 2 (x-tilt), 3 (y-tilt), 4 (defocus), 5 (astigmatism), 6 (astigmatism), 7 (y-coma), 8 (x-coma), and 11 (spherical aberration). On the basis of Eq. (11), together with Eqs. (12) and (13), using the values of $\langle b_j^* b_{j'} \rangle$ calculated from Eq. (15), we numerically evaluated the degree of coherence of the resultant field for some selected values of σ_ϕ/R under the condition of $\phi_0 = \pi$. The results are illustrated in Fig. 2.

It is clear from these results that low-order adaptive optics significantly enhances the spatial coherence for $\sigma_\phi/R = 1$, while its effect is less significant for $\sigma_\phi/R = 1/4$. This behaviour may be intuitively understood in terms of the Zernike-mode decomposition of the initial field: the number of the Zernike modes decomposed is small for large σ_ϕ/R, i.e., when the spatial coherence of the initial field is high, while it is large for small σ_ϕ/R, i.e., when the spatial coherence of the initial field is low. Hence, it is readily expected that the effect of the low-order Zernike-mode correction on the enhancement of the spatial coherence of the resultant field becomes stronger as the number of the Zernike modes existing in the initial field decreases or, equivalently, as the spatial coherence of the initially partially coherent field to be corrected increases.

This conclusion is well known in the astronomical context of adaptive optics, where the point spread function, Strehl ratio and wave-front variance are the usual metrics, as shown by Noll.[9] However, we emphasize that adaptive optical systems can be used as a modifier or filter of spatial coherence, provided that an initially spatially partially coherent field to be modified is given in advance.

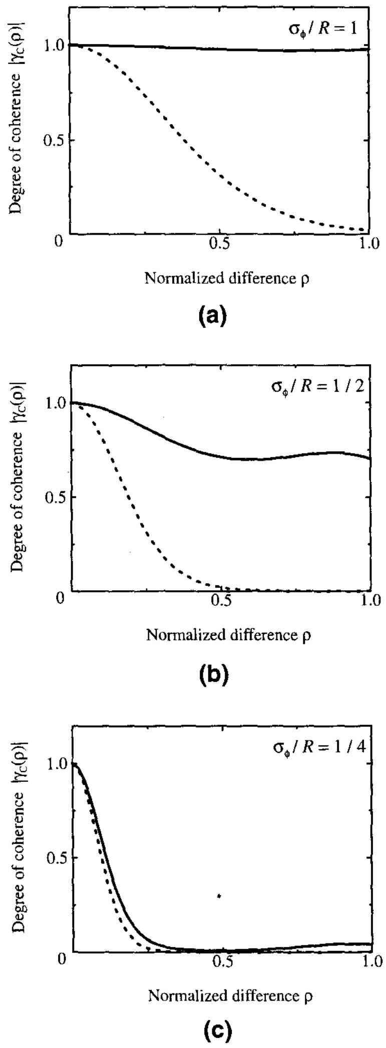

Fig. 2. Degree of coherence of the resultant field for $\sigma_\phi/R = 1$ (a), 1/2 (b), and 1/4 (c). The dotted curves indicate the degree of coherence of the resultant field without correction, calculated from Eq. (16).

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