Compact multireference wavefront sensor design

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We present a compact optical design for a multireference Shack–Hartmann-based wavefront sensor (WFS) for multiconjugate adaptive optical systems. The key component of this WFS design is a field lenslet array that separates the exit pupil images in the sensing plane for all reference sources. An analytical method for WFS optical design is presented, and the optimal strategy for selecting optical components from a discrete set is outlined. The feasibility of the WFS design has been demonstrated for a prototype WFS system in a laboratory setup with five reference sources and two deformable mirrors representing a wavefront-distorting medium. © 2005 Optical Society of America

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Imaging through a phase-distorting medium can be improved for a small field of view by use of an adaptive optical (AO) system working with a single reference light source. The correction of the imaging beam is achieved by adjustment of the figure of a deformable mirror (DM) to null the residual phase errors as measured by a wavefront sensor (WFS). In general, the phase correction over a larger field of view than that for single DM correction on axis can be achieved with a multiconjugate AO system operating with multiple reference sources and several WFSs and DMs. In astronomical AO applications, these reference sources are referred to as natural or laser guide stars, depending on their origin. For wavefront sensing in the eye, a scanner may be used to produce an array of laser spots on the retina.

Combining all reference beams in a single detector is a practical way to reduce the size and complexity of the WFS system if it is done by means of a single optical element. We propose to use a field lenslet (FL) array with a regular array of reference sources for multiconjugate AO correction in the eye and astronomical telescopes. The concept of the FL array is depicted in Fig. 1. The FL array is located at intermediate imaging plane I, so each image of the reference source falls onto a separate lenslet, which reimages the entrance pupil at sensing plane P; If the FL were removed, all reference sources would have their common exit pupils on plane P'.

For a given imaging telescope system (TEL) with entrance pupil diameter D, focal length f_TEL, and number of reference sources N_RS, we estimated the following parameters of the multireference WFS: single lenslet diameters d_FL and d_SH, effective diameters D_FL and D_SH (defined as a maximum chief ray separation for marginal sources), and focal lengths f_FL and f_SH for the field and Shack–Hartmann (SH) lenslet arrays, respectively; collimator focal length f_COL, and axial shift t of the exit pupil, as shown in Fig. 1.

A wavefront aberration W measured in a SH subaperture d is an averaged wavefront tilt \( \alpha \), which can be expressed as a phase difference in radians:

\[
W = 2\pi d \alpha / \lambda. \tag{1}
\]

Therefore wavefront sensing of atmospheric turbulence with correlation length \( r_0 \) is usually performed with a number of SH lenslets per pupil diameter greater than \( N_{SH} = D/r_0 \). Using \( m \times m \) pixels in a CCD subframe for each subaperture and sampling the spot width by \( n \times n \) pixels, one can easily show from Eq. (1) that the maximum wavefront aberration for which the spot starts crossing the edge of its own subframe is \( W_{max} = \pi (m-n)/n \). For given \( n \) and \( W_{max} \), the minimum subframe size is

\[ m = n(1 + W_{max}/\pi). \tag{2} \]

According to Kolmogorov turbulence models, the angle of arrival has a Gaussian distribution. The standard deviation of the wavefront tilt angle (in radians) over an aperture of size \( d \) is \( \sigma_\alpha = 0.427(d/r_0)^{5/6}(\lambda/d) \). Peak wavefront excursions with tilt angle values \( |\alpha_{max}| > 2.5\sigma_\alpha \) occur with 0.6% probability. For this maximum wavefront tilt, \( \alpha_{max} \) over a subaperture \( d = r_0 \), from Eq. (1) we find that \( W_{max} = 2.14\pi \). Using Eq. (2), we estimate the subframe size, covering the entire angle-of-arrival range \( \pm \alpha_{max} \), \( m = n \), where by Nyquist’s sampling criterion \( n = 2 \).

In general, for a known range \( \pm \alpha_{max} \) over a subaperture \( d = D/N_{SH} \), we need \( m = (1 + 2\alpha_{max}d/\lambda) \).

It is a common practice to couple the SH lenslet array with a CCD detector by means of an optical relay system if the lenslet’s focal length \( f_SH \) is too short for direct imaging onto the CCD. The relay system is also helpful for matching the size of lenslet \( d_{SH} \) to the...
integer number of pixels across the subframe. For optimal coupling, the relay system’s magnification is found:
\[ \gamma = mp_{\text{CCD}}/d_{\text{SH}}, \]  
(3)
where \( p_{\text{CCD}} \) is a detector pixel’s size. Forming the spot width of \( np_{\text{CCD}} \) in the subframe requires lenslet focal length \( f_{\text{SH}} = np_{\text{CCD}}d_{\text{SH}}/\lambda \gamma \), and from Eqs. (2) and (3) we get
\[ f_{\text{SH}} = \gamma d_{\text{SH}}^2/\lambda (W_{\text{max}} + \pi). \]  
(4)
At the stage of practical achievement of the WFS system, one could use optical components with only discrete values of \( d_{\text{SH}}, f_{\text{SH}}, \) and \( p_{\text{CCD}} \). Therefore, from Eq. (2), one should first find an integer \( m \) for which \( W_{\text{max}} \geq 2.14 \pi \) and \( 1 < n \leq 2 \) and then, using Eq. (3), select values for \( p_{\text{CCD}} \) and \( d_{\text{SH}} \) such that magnification \( \gamma = 1 \). Finally, in choosing focal length \( f_{\text{SH}} \) it is advisable to aim at the value given by Eq. (4) or smaller. If the available SH lenslet array has a significantly shorter focal length \( f'_{\text{SH}} \), one should recalculate \( W_{\text{max}} \) from Eq. (4) and estimate \( m \) for a new value of \( W'_{\text{max}} \) from Eq. (2). As a result, the WFS working range will be larger at the expense of increasing \( m \) and the total number of pixels across a CCD detector, which is \( N_{\text{CCD}} = (mN_{\text{SH}} + k_{\text{b}})N_{\text{RS}} \), where \( k_{\text{b}} \) is the number of boundary pixels between marginal subframes of two adjacent groups of SH spots associated with two adjacent sources. To separate these groups by \( k_{\text{b}} \), one has to fulfill the pupil separation condition for the axial shift:
\[ t = f_{\text{SH}} + f_{\text{COL}}N_{\text{CCD}}p_{\text{CCD}}/(\gamma d_{\text{FL}}). \]  
(5)
The exit pupil’s position is defined by axial shift \( t = f^2_{\text{COL}}[1/(f_{\text{FL}}) + (l-f_{\text{TEL}})/(f^2_{\text{TEL}})]. \) Assuming a telecentric path for the beams exiting the TEL, that is, \( l = f_{\text{TEL}} \), and expressing the FL array’s effective diameter as \( D_{\text{FL}} = D_{\text{COL}} = k d_{\text{FL}}(N_{\text{RS}} - 1), \) where \( k \) is the number of lenslets between adjacent images in focal plane 1, from Eq. (5) we derive a relationship for selecting \( f_{\text{FL}} \) for a given total number of pixels across the CCD detector:
\[ N_{\text{CCD}} = mD_{\text{FL}}[f^2_{\text{COL}} - f_{\text{FL}}f_{\text{SH}}]/(f_{\text{FL}}f_{\text{COL}}d_{\text{SH}}), \]  
(6)
where \( f_{\text{COL}} \) and \( d_{\text{FL}} \) are chosen to match marginal field angle \( \omega = d_{\text{SH}}N_{\text{SH}}D_{\text{FL}}/(2D_{\text{COL}}). \) Finally, we estimate the imaging system’s focal length \( f_{\text{TEL}} = D_{\text{FL}}/2\omega, \) axial shift \( t = f^2_{\text{COL}}/f_{\text{FL}}, \) and the SH lenslet array’s effective diameter \( D_{\text{SH}} = f_{\text{COL}}D_{\text{FL}}/f_{\text{FL}}. \)
In a real WFS system, parameter \( t \) is used to make fine adjustments for lateral separation of the pupils and groups of SH spots. To do this, one alters the value \( f_{\text{SH}} \) by a small amount \( \Delta f_{\text{SH}} \) to match the discrete value \( f_{\text{FL}} \) to the number \( N_{\text{CCD}} \) in Eq. (6) and then recalculates \( t \) from Eq. (5). One may also adjust the pupil separation by slightly varying positions of the sources in object space. It is worth noting that Eq. (5) holds even for \( l < f_{\text{TEL}}; \) thus one can make the WFS system more compact along the optical axis by abandoning the telecentric beam path.
To verify the concept of the multireference WFS, a prototype WFS system has been built and integrated into a laboratory optical setup and tested with five reference sources that form a regular cross and two DMs that represent a wavefront-distorting medium. Table 1 lists values for the major parameters of WFS in the order of their selection or estimation. We have \( D = 8.2 \text{ mm}, N_{\text{RS}} = 3, \) and \( \lambda = 0.589 \mu \text{m}. \)

<table>
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<th>Parameters of choice values³</th>
<th>( d_{\text{SH}} )</th>
<th>( f_{\text{SH}} )</th>
<th>( m )</th>
<th>( p_{\text{CCD}} )</th>
<th>( N_{\text{SH}} )</th>
<th>( k_{\text{b}} )</th>
<th>( d_{\text{FL}} )</th>
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<td>Estimated parameters values³</td>
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<td>( n )</td>
<td>( \gamma )</td>
<td>( N_{\text{CCD}} )</td>
<td>( D_{\text{FL}} )</td>
<td>( \omega )</td>
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<td>0</td>
<td>1.75</td>
<td>3</td>
<td>48</td>
<td>91</td>
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³Length units are millimeters.

![Fig. 2. SH spots centered in subframes.](image1)
![Fig. 3. Optical layout of the multireference WFS.](image2)
tuator OKO mirror placed in front of the pupil at a
distance \( h = 70 \) mm. The effective working diameters
of DM1 and DM2 are 8 and 21 mm, respectively.
Thus DM1's size matches that of the pupil well,
whereas DM2 requires an afocal system with beam
expansion \( \varepsilon = 1.6 \) to match the metapupil diameter
\( D + 2\omega h = 13.8 \) mm.

To induce wavefront aberrations in the 5 probing
beams, we applied maximum voltage to all 37 actuators
of DM1. As a result, the membrane of DM1 assumed
a concave shape, causing defocus in all five beams,
as shown in Fig. 4(a), where SH spots are displayed
together with SH vectors indicating directions
and amount of spot displacements. The defocus term
measured in the central beam does not differ much
from defocus seen in the off-axis beams, indicating
that DM1 is located in the pupil plane. Figure 4(a)
also reveals that the DM1 vertex is centered by one
subframe (0.75 mm at the DM1 plane).

To estimate the amount of defocus introduced by
DM1, we measured the positions of 24 marginal SH
spots located an average distance \( L_\alpha = 1 \) mm (five sub-
frames) from the beam center. We found that their
average displacement toward the center is \( \Delta_\alpha = 24.2 \) \( \mu \)m (two pixels). In these subapertures, an
average wavefront tilt is \( \alpha = \Delta_\alpha / \gamma_{SH} \); therefore, at
the pupil plane, the radii of curvature of the wavefront
and of DM1 are \( \alpha = R_{WF} = L_\alpha f_{TEL} / (f_{COL})^2 \approx 3.5 \) m and
\( R_{DM1} = 7 \) m, respectively. The DM1 stroke achieved
was \( D^2 / 8R_{DM1} = 1.2 \) \( \mu \)m, which agrees with measurements
obtained with a commercial Twyman–Green interferometer.

Figure 4(b) shows SH spots and vectors when half-
maximum voltage was applied to all 59 actuators of
DM2. Apart from the visible defocus term, there is
also clear tilt component for all five beams. The tilt in
the central beam is due to DM2 vertex centering by
three subframes (3.6 mm at the DM2 plane). The tilt
for off-axis beams is different, indicating that these
beams are separated from the central beam by
roughly four subframes (\( \delta = 4.8 \) mm on DM2), which
leads to an estimated distance of DM2 from the pupil
plane of \( h = \delta / \omega e = 74 \) mm. Similarly to the case with
DM1, from the amount of defocus measured in the
central beam we found that \( R_{DM2} = 17.8 \) m, which
responds to a 3.1 \( \mu \)m DM2 surface stroke for a
21 mm effective diameter, whereas measurements
with an interferometer revealed a 4 \( \mu \)m peak-to-
valley surface deviation. We conclude that our experi-
mental results show a successful wavefront sensing
of aberrations.

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