Double-pass axially resolved confocal Mueller matrix imaging polarimetry

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We present experimental depth-resolved complete polarization-sensitive measurements of a stack of linear retarders and glass plates by using what is to the best of our knowledge the first combination of a confocal imaging system with a complete Mueller matrix polarimeter. The axially resolved Mueller matrices were compared with a forward simulation, with good agreement. © 2005 Optical Society of America

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Recent developments in polarization-sensitive imaging have given rise to polarization-sensitive optical coherence tomography (PS-OCT)\(^1\) systems. PS-OCT is an incomplete polarimetry technique with a powerful depth-sectioning capability. The results have been commonly stated in terms of Stokes vectors and Mueller matrices, but PS-OCT, as it has been implemented until now,\(^2\) is capable of assessing the state of polarization of only the portion of light that is totally polarized. In other words, with PS-OCT, only a subset of the 16 Mueller matrix coefficients has been measured. The definition of the Stokes vector in terms of irradiances makes the Mueller calculus most suitable for describing irradiance-measuring instruments.\(^3\) Incomplete interpretations of polarization-dependent features may arise if the limitations of the measuring instrument are not taken into account. Incomplete polarimeters that are not able to measure depolarization, for instance, may acquire incorrect retardance measurements of a depolarizing sample.

A Mueller matrix is a 4 by 4 matrix that represents the effect of a sample on the state of polarization of light as a linear operator that acts on a Stokes vector. The Mueller calculus can account for partially polarized incident beams and can handle the propagation through depolarizing optical systems. The 16 elements of a Mueller matrix are, in general, linearly independent. Thus, if no assumptions are made about the effect of a sample on the state of polarization of light, and if a complete characterization of the specimen is required, the 16 Mueller matrix elements need to be measured individually. Previously these 16 Mueller coefficients were measured only with two-dimensional imaging techniques.

In this Letter we present axially resolved complete Mueller matrix measurements of an artificially built stack of linear retarders and glass plates. Figure 1 shows a schematic diagram of the confocal Mueller matrix imaging polarimeter. A concise description of the technique that we introduce and the instrument that we developed can be found in Ref. 5. In brief, two Pockels cells modulated the state of polarization of the light (\(\lambda=532\) nm) incident on the sample, and a division of amplitude polarimeter analyzed the light scattered from the sample after it had passed through the confocal optics. To the best of our knowledge this constitutes the first combination of a three-dimensional imaging technique with a complete Mueller matrix polarimeter. For the calibration of the double-pass measurements, we implemented a modified version of the eigenvalue calibration method described previously by Compain et al.\(^6\) The modified method, which is very robust and is applicable to any double-pass polarimeter, is also described in Ref. 5.

A stack of three linear 560 nm quarter-wave plates made from cellulose acetate butyrate (Edmund Scientific N53-205) was placed between two microscope glass slides. The two outer retarders of the stack, which were cut from the same acetate sheet and were aligned along the cut side, were oriented horizontally. The middle retarder was oriented at approximately 45°. The elements of the stack were not cemented; we kept them together by pressing the two glass slides against each other. No mechanical mount was used to fine tune the azimuth alignment of the retarders, but, once the stack was placed on the holder, the angle did not change. The exact azimuth angles were calculated from the experimental Mueller matrices.

The stack of retarders was moved along the optical axis, in steps of 10 \(\mu\)m, by a manual micrometer.
screw, toward the objective lens (Obj1; N.A., 0.14) while three Mueller matrices were measured at each axial position. Each Mueller matrix was acquired in 51.2 ms. The calibrated Mueller matrix measurements are presented in Fig. 2. The peaks in the graph of coefficient \( \text{retStack}_{11} \), labeled A–F, correspond to the positions of the interfaces between the elements of the stack. The three measurements at each axial position were averaged; the standard deviation was always smaller than the marker size in the figures.

The double-pass calibration measurements of the polarimeter were taken with the confocal pinhole inserted in the system in exactly the same axial position as when the axial scans were made, and without objective lens Obj1. It was stated in a companion paper that the removal of the objective Obj1 during the calibration of the polarimeter introduced an error no larger than 3% in the off-diagonal Mueller matrix coefficients of a dielectric mirror. The percentage was calculated with respect to the coefficient with the highest value on the diagonal. For the purpose of this study, this error was considered small.

In an attempt to eliminate calibration artifacts derived from errors in the axial position of the pinhole, we used the stack Mueller matrices of the three axial positions with the highest signal to represent each interface. For example, the Mueller matrices of positions 0.58, 0.59, and 0.60 mm were used to represent the interface B. These three matrices were first normalized to unit reflectance for nonpolarized light (coefficient \( \text{retStack}_{11} \)) and then averaged. The root-mean-square (rms) values of the 16 standard deviations of each surface were only slightly larger than those obtained in the repeatability tests (see Ref 5), i.e., A, 1.5%; B, 1.5%; C, 5.4%; D, 2.5%; E, 3.6%; and F, 1.9%.

Simulated Mueller matrices were fitted to the azimuth orientations of the three linear retarders within the stack and compared to the experimentally measured matrices. The first two simulated interfaces (A and B) were assumed to be equal to the identity matrix. The incidence of the light on the glass plate interfaces was approximately perpendicular, and no phase shift was expected from the reflection on the dielectric surfaces. The simulated Mueller matrices of the rest of the interfaces, C–F, were calculated by use of a retardance of 560/4 nm for each pass through a plastic retarder (V). The simulated double-pass Mueller matrix of interface C, for ex-

![Fig. 2. Axial Mueller matrix scan with a 5 \( \mu \)m (4 optical units radius) confocal pinhole of the stack of retarders. The scale for the off-diagonal elements is three times smaller than for the diagonal elements.](image-url)
ample, as a function of the fitted azimuth angle of the front retarder ($\theta_1$), was

$$C_{\theta_1} = R(-\theta_1)VR(\theta_1)VR(-\theta_1),$$

where $R(\theta)$ is the matrix that rotates a Stokes vector by azimuth angle $\theta$ about the $z$ axis and $E$ is the Mueller matrix of an ideal mirror. The mirror matrix denoted at the left in Eq. (1) ensured that the azimuth of the double-pass matrix was measured in the coordinate system of the first pass and the mirror matrix denoted in the middle part of the equation separated the first pass from the second pass. It was assumed for Eq. (1) that the behavior of the retarder was the same in forward and backward propagation. The azimuth angle of the simulated sample was found numerically as the angle that minimized the rms of the 16 coefficients of the difference between the simulated and the experimental matrices; the minimum rms errors between these matrices were as follows: A, 3.4%; B, 4.8%; C, 11.6%; D, 3.6%; E, 3.3%; and F, 5.1%. Once $\theta_1(=5.0^\circ)$ was found, the same procedure was used to calculate $\theta_2=43.9^\circ$, using the azimuth of the experimental Mueller matrix of interface D and keeping $\theta_1$ constant. Similarly, $\theta_3=5.3^\circ$ was found by use of the experimental Mueller matrix of the interface E, with $\theta_1$ and $\theta_2$ kept fixed.

No account of the axial component of the electric vector introduced by the focusing of the light was taken in this forward simulation, and the retarders were assumed to introduce the exact retardance specified by the manufacturer. This simple model showed good agreement with the experimental results. For systems with larger N.A., however, the effect of the axial component may need to be taken into account.

In addition to the rms values presented above, the polar decomposition of Lu and Chipman was performed on all the experimental and simulated matrices; the calculated retardance and angle of linear retardance (azimuth) are presented in Fig. 3. The shaded area in Fig. 3(b) is a reminder that the simulated angle of linear retardance of interfaces A and B was undefined; since both interfaces were ideally represented by the identity matrix, the retardance was equal to zero. However, the experimental retardance vector was normalized to unity before the angle of linear retardance was calculated. The retardance value of interface C was statistically different from the simulated value. It is worth noting that the signal recorded from this interface was the smallest of the six interfaces. The depolarization power at the interfaces was also calculated, and the values were small: The largest value obtained was $0.09\pm0.03$ for interface D. The measurements taken from the stack of retarders were considered specular reflections from the interfaces and no depolarization was expected, which agreed with the results obtained.

As a first approximation, the forward simulation presented here agreed well with the experimental results. The results show that it is possible to obtain confocal depth-resolved complete polarization-sensitive measurements. More work needs to be done toward the extension of the technique to systems with higher N.A., and the effect of the confocal system on the polarimetry lateral resolution remains to be studied further. Contrary to results for polarization-sensitive optical coherence tomography systems, for which the measurement of Mueller matrices and Stokes vectors at different depths has been reported, in the study reported here the 16 elements of the Mueller matrix were measured independently. This means that the measurements presented here were not Jones matrices converted to Mueller matrices; they were complete Mueller matrices that can include the depolarization information. This constitutes what is to our knowledge a first-time achievement and is the central part of this Letter.

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References