Pupil matching of Zernike aberrations

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Abstract: The measurement of the wavefront at the exit pupil of an optical system is a reliable method to investigate its imaging performance. It is sometimes necessary to compare the measurement obtained with a wavefront sensor to some known aberration function, which is for example given by a simulation or a gold standard measurement technique such as interferometry. For accurate measurement systems, the residual after direct subtraction of the two wavefronts is often partly due to a mismatch between coordinate systems. We present in this paper a method that uses the Zernike expansion of wavefronts to numerically cancel such small alignment errors. We use this algorithm to quantify the accuracy of a custom-built Shack-Hartmann wavefront sensor in the measurement of known aberration functions.

OCIS codes: (220.0220) Optical design and fabrication; (120.3940) Metrology; (010.7350) Wave-front sensing; (120.4800) Optical standards and testing.

References and links

1. V. N. Mahajan, "Zernike Polynomial and Wavefront Fitting," in *Optical Shop Testing*, D. Malacara, ed. (Wiley-Interscience, 2007), pp. 498–546.
2. J. Pfund, N. Lindlein, and J. Schwider, "Misalignment effects of the Shack–Hartmann sensor," Appl.Opt. **37,** 22–27 (1998).
3. D. R. Neal, J. Copland, and D. Neal, "Shack–Hartmann wavefront sensor precision and accuracy," Proc. SPIE **4779,** 148–160 (2002).
4. P. Rodrguez, R. Navarro, J. Arines, and S. Bara, "A new calibration set of phase plates for ocular aberrometers," J. Refract. Surg. **22,** 275–84 (2006).
5. K. M. Morzinski, K. B. W. Harpsoe, D. T. Gavel, and S. M. Ammons, "The open-loop control of MEMS: modeling and experimental results," Proc. SPIE **6467,** 64670G (2007).
6. L. Lundstrom and P. Unsbo, "Transformation of Zernike coefficients: scaled, translated, and rotated wavefronts with circular and elliptical pupils," J. Opt. Soc. Am. A **24,** 569–577 (2007).
7. S. Bara, J. Arines, J. Ares, and P. Prado, "Direct transformation of Zernike eye aberration coefficients between scaled, rotated, and/or displaced pupils," J. Opt. Soc. Am. A **23,** 2061–2066 (2006).
8. R. Navarro, E. Moreno-Barriuso, S. Bara, and T. Mancebo, "Phase plates for wave-aberration compensation in the human eye," Opt. Lett. **25,** 236–238 (2000).
9. H. Schreiber and J. H. Bruning, "Phase Shifting Interferometry," in *Optical Shop Testing*, D. Malacara, ed. (Wiley-Interscience, 2007), pp. 547–666.

1. Introduction

There are many examples in the optical workshop for which one wants to compare a wavefront measurement to a prediction or a measurement obtained with a different technique. In many cases, the mismatch between the two wavefronts is mainly due to alignment artefacts in the measurement plane. Translational and rotational mismatches of coordinate systems make the direct comparison of Zernike aberrations difficult and limit the benefit of their mathematical

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properties to report wavefront measurements [1]. The method that we present in this paper uses the Zernike description of the two wavefronts that we want to compare, and retrieves the geometrical parameters that minimises the Root Mean Square (RMS) value of their difference.

We developed this method to cancel alignment artefacts that arise when assessing the accuracy of a custom-built Shack-Hartmann Wavefront Sensor (SHWFS) with the measurements of known aberration functions. It is well established that the accuracy of a SHWFS mainly depends on the alignment of the array of lenslets with respect to the imaging detector [2]. The corresponding distance sets the sensitivity of the SHWFS, and should therefore be calibrated. The sampling and the processing of each Shack-Hartmann spot also have a fundamental influence on the linearity of a SHWFS [3]. The accuracy of a SHWFS is classically investigated using spherical wavefronts emitted by a translated point source, although it is more reliable to test a measurement system with known aberration functions that are typical of the targeted application [4]. Another possible application of this method is the calibration of open-loop adaptive optics systems [5], as it can match the simulation of the response of the deformable mirror to the wavefront sensor.

2. Principle of the algorithm

We assume that we have previously obtained two sets of Zernike coefficients ($\mathbf{z}_0$ and $\mathbf{z}_{ref}$) that describe the same optical system in two slightly different coordinate systems. We want to transform $\mathbf{z}_0$ to $\mathbf{z}_1$, so that $\mathbf{z}_1$ and $\mathbf{z}_{ref}$ describe the optical system in the same coordinate system. To do so, we search the geometrical parameters $\mathbf{p}$ that relate the two coordinate systems such that the norm of the residual $\|\mathbf{z}_1 - \mathbf{z}_{ref}\|$ is minimised. This is equivalent to minimising the RMS error between the two wavefronts, as Zernike coefficients are orthonormal over the circular measurement pupil:

$$\widehat{\mathbf{p}} = \text{argmin}\left\{\|\mathbf{z}_1 - \mathbf{z}_{ref}\|\right\} \tag{1}$$

$\mathbf{z}_1$ is implicitly dependant on $\mathbf{p}$. The components of $\mathbf{p}$ are: $[t_x, t_y, \theta, \gamma]$ for the translation (t_x, t_y), rotation (θ) and scale (γ). We consider in this paper sets of 63 Zernike coefficients (up to the tenth radial order, piston and tilts excluded). The forward problem of computing $\mathbf{z}_1$ for a given set of parameters $\mathbf{p}$ has been previously described [6, 7]. We adopt the fully numerical method of Reference [7] that consists in performing the least-square fit of a wavefront map with a variable coordinate system. For two given coordinate systems that are related by the parameter $\mathbf{p}$, there is a linear relationship between Zernike coefficients:

$$\mathbf{z_1} = \mathbf{z}_0 + \mathbf{M_p} \times \mathbf{z}_0 \tag{2}$$

In general, the components of the matrix $\mathbf{M_p}$ do not vary linearly with $\mathbf{p}$. However, we write a linear approximation that is valid for small misalignments $\mathbf{p}$. This approximation uses four elementary transformations of pure horizontal translation, vertical translation, rotation and scale:

$$\mathbf{M_p} \simeq \frac{t_x}{t_{x,0}} \times \mathbf{M}_{[t_{x,0},0,0,1]} + \frac{t_y}{t_{y,0}} \times \mathbf{M}_{[0,t_{y,0},0,1]} + \frac{\theta}{\theta_0} \times \mathbf{M}_{[0,0,\theta_0,1]} + \frac{\gamma - 1}{\gamma_0 - 1} \times \mathbf{M}_{[0,0,0,\gamma_0]} \tag{3}$$

The values of $t_{x,0}$, $t_{y,0}$, θ_0 and γ_0 are fixed, and should correspond to typical misalignments. The elementary matrices are normalised by those values. Iterations of the algorithm make the accuracy of our method relatively insensitive to the choice of these values. Each matrix of Eq. (3) is of dimension 63×63, and can be multiplied by the vector $\mathbf{z_0}$. We obtain four column vectors of dimension 63×1 that we concatenate to build a matrix $\mathbf{A}$, which we compute only once. We write the linear model that relates $\mathbf{z_0}$, $\mathbf{z}_{ref}$, and $\mathbf{p}$:

$$\mathbf{A} \times \mathbf{p} = \mathbf{z_0} - \mathbf{z}_{ref} \tag{4}$$

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The linear model of Eq. (3) is the approximation that we use to iteratively solve Eq. (1):

$$\mathbf{p}_{i+1} = \mathbf{p}_i + \mathbf{A}^+ \times (\mathbf{z}_i - \mathbf{z}_{ref}) \text{ , with: } \mathbf{z}_i = \mathbf{z}_0 + \mathbf{M}_{\mathbf{p}_i} \times \mathbf{z}_0 \quad (5)$$

Where $\mathbf{A}^+$ stands for the pseudo-inverse of the matrix $\mathbf{A}$. The iterations of Eq. (5) are initialised with $\mathbf{p}_{i=0} = [0,0,0,1]$ and $\mathbf{z}_{i=0} = \mathbf{z}_0$. At each iteration, the matrix $\mathbf{M}_{\mathbf{p}_i}$ is computed using the numerical method of Reference [7]. The algorithm stops when the norm of the difference between two iterations, $\|\mathbf{z}_i - \mathbf{z}_{i-1}\|$, is smaller than 1 nanometer. Intuitively, the algorithm converges to a small residual $\|\mathbf{z}_1 - \mathbf{z}_{ref}\|$ if the initial alignment errors are small enough. If the reference map described by $\mathbf{z}_0$ has some circular symmetry, the rotation angle θ should not be estimated. The $\mathbf{A}$ matrix is in this case built from three terms only in Eq. (3). Our method is a simplification of Newton's method for finding the zero of a function, which is in our application the RMS error. Instead of computing the derivative of this function, our model relies on a simplistic linear approximation that is not updated at each iteration.

3. Description of the experiment

3.1. Methods

We have used a commercial Twyman-Green Phase Shifting Interferometer (PSI) as a reference to test our custom-built SHWFS. The PSI is a built-in system developed by Fisba Optik, with a single optical head for emission and detection of a 632.8 nm laser light source. We illustrate in Figure 1 the optical layout of the measurements. The relay lens R is an auxiliary optic also manufactured by Fisba Optik, which performs a calibrated magnification of the measurement plane. The measured pupil has a known diameter of 5.25 mm in the measurement plane. The wavefront maps are exported from the commercial software as 500×500 matrices and processed using MATLAB. The measurement pupil is circular, and has a 500 pixel diameter. The custom-built SHWFS consists of a lenslet array of square pitch (0.2 mm), with a calibrated effective focal length of 7.15 ± 0.01 mm. The measured pupil is expected to be around 6 mm in diameter, but the magnification of the lenslet array to the measurement plane was not accurately calibrated. The SHWFS has an auxiliary camera that images the measurement plane for a careful alignment. The ensemble (SHWFS + relay + auxiliary camera) is physically translated with a 10 μm resolution for pupil alignment. We have measured four phase plates that introduce Zernike aberrations of low radial order, with a high optical quality [8]. For both measurement systems, a reference wavefront is measured by simply removing the plate. The plates have some marks that we use for a careful alignment with respect to the measurement systems, which is a typical approach for testing optics with an interferometer [9].

The sensitivity of the SHWFS has been calibrated with a translated point source (the output of a monomode fibre). In the same experiment, we quantify the accuracy of the SHWFS using the mean variation of the tilt-removed wavefronts. We evaluate the corresponding mean RMS error over a $\pm 0.5^\circ$ field of view as 11 nm (using 63 Zernike coefficients, tilts excluded). This non-linearity comes from the light-insensitive region of the pixels of the SHWFS detector, and is the main source of inaccuracy for the measurement of static aberrations. We experimentally evaluate the precision of the measurement by recording a continuous series of 600 measurements. We found a precision of 2 nm RMS (63 Zernike coefficients), which is a combination of electronic and photon noise. The PSI does not suffer from any significant source of inaccuracy, and its manufacturer guarantees a precision in the order of 5 nm. In order to operate the PSI correctly, it is important to perform a calibration of the phase-shifting piezo-mirror and to perform the measurements in a vibration and turbulence free environment.

With the PSI, we numerically defined the measurement pupil directly on the interferograms. The PSI measurement diameter is smaller than the SHWFS measurement diameter, so that the

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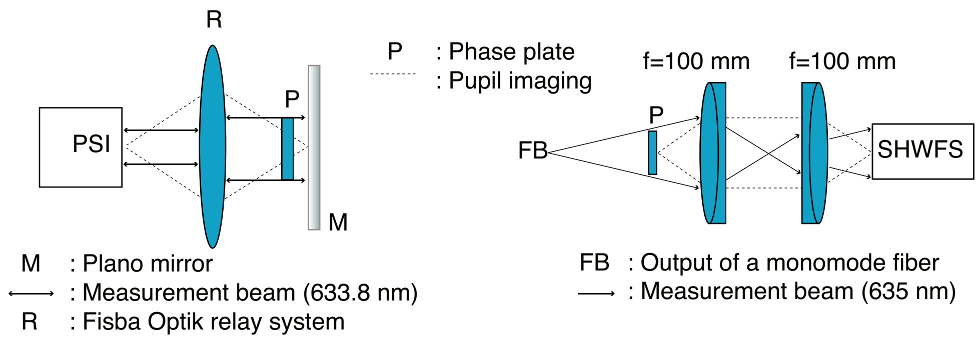

Fig. 1. Optical layout of the measurements. Left: double-pass measurement with the commercial PSI. Right: single-pass measurement with the SHWFS.

PSI pupil is entirely contained in the SHWFS pupil for translations up to around 0.35 mm. the pupil over which we want to compare the two wavefronts should be entirely contained by the original pupil. This ensures that no inaccurate data extrapolation is performed during the pupil transformation. To define the linear approximation of Eq. (3), we chose values $t_{x,0} = t_{y,0} =$ 0.21 mm, $\theta_0 = 1^\circ$, and $\gamma_0 = 1.05$.

Figure 2 illustrates the task of our algorithm. The left graph shows the wavefront map obtained with the PSI, over a calibrated 5.25 mm pupil. The middle graph shows the map obtained by the SHWFS in its own coordinate system ($\simeq$ 6 mm diameter), and the output of our algorithm (the coordinate system over which the SHWFS map fits the PSI maps). The right graph represents the SHWFS map in the adjusted coordinate system, which is similar to the PSI map. The RMS error of the difference is 11 nm.

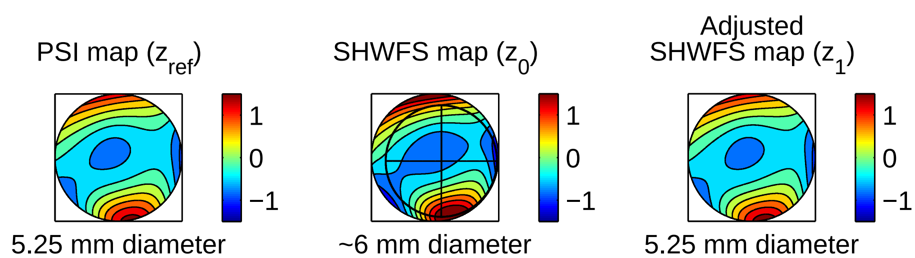

Fig. 2. Wavefront maps in microns, as measured by the PSI (left) and the SHWFS (middle). Right: Wavefront map as measured by the SHWFS, after correction of the pupil mismatch.

3.2. *Results*

Table 1 shows the results obtained for the four phase plates. The algorithm decreases the mean RMS of the difference between the SHWFS and the PSI measurements from around 170 nm ($\mathrm{RMS}_0 = \|\mathbf{z}_0 - \mathbf{z}_{ref}\|$) down to typically 11 nm ($\mathrm{RMS}_1 = \|\mathbf{z}_1 - \mathbf{z}_{ref}\|$). When correcting for the scale only, the mean RMS is around 150 nm. Typically, around 10 iterations are required to reach the convergence criterion of 1 nm. The largest relative variation of the estimated magnification factor γ is 0.8%, which shows that our algorithm has some consistency from one plate to the other. Figure 3 compares the sets of Zernike coefficients $\mathbf{z}_{ref}$, $\mathbf{z}_0$ and $\mathbf{z}_1$.

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Table 1. Experimental results

	RMS_0 (nm)	RMS_1 (nm)	t_x (mm)	t_y (mm)	θ (degree)	γ
Plate 1	188	11	0.30	-0.17	-0.08	1.144
Plate 2	169	11	0.28	-0.23	-2.88	1.147
Plate 3	109	6	0.12	0.02	-1.07	1.154
Plate 4	200	17	0.08	-0.29	-2.60	1.152

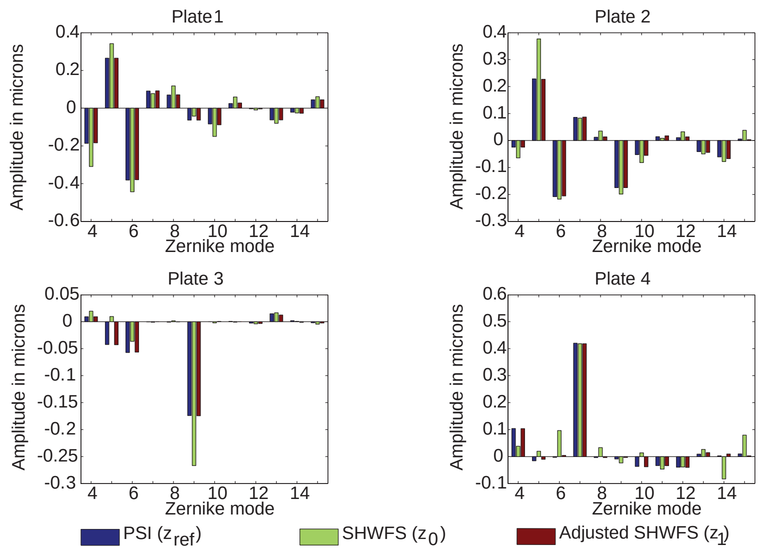

Fig. 3. Comparison of the measured Zernike aberrations, for the four phase plates. The red bars show the SHWFS measurement after pupil correction, and are very comparable to the PSI measurements (blue bars).

We have investigated the linearity in the estimated translation and rotation of the pupil, over a range that seems suitable for optimising an alignment procedure ($\theta = \pm 2^\circ$, $t_x, t_y = \pm 0.15$ mm). To do so, we computed the Zernike coefficients $\mathbf{z}_{ref}$ over translated and rotated coordinate systems. This was done as a first preprocessing step, before invoking our algorithm. As shown in Figure 4, we have imported the wavefronts measured by the PSI over a larger pupil to allow for translations of the pupil without vignetting the PSI wavefront map (600×600 pixels instead of 500×500). We expected our algorithm to retrieve the numerically-induced shifts and rotations, with an offset that corresponds to the values of Table 1.

First we introduced shifts of the pupil only, for each plate. We show in Figure 5 the estimated positions of the centre of the pupil, for the first plate (left graph). A larger inaccuracy in the estimated translations appears for $X \simeq 0.15$ mm (right side of the graph), and is most probably related to the original shift between the two measurements ($t_x = 0.3$ mm, in Table 1). In this case, the actual shift between the two coordinate systems is therefore approximatively 0.45 mm, and the PSI coordinate system is not entirely contained in the original SHWFS wavefront map. For the first plate, the root mean square error in the estimated positions is 5 microns. We found similar values for the other plates (12, 7, and 8 microns for the plates 2, 3, and 4).

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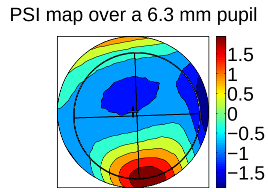

Fig. 4. PSI map of the first plate, measured over a 6.3 mm pupil diameter. To obtain the results of Figure 5, we have fitted the set of Zernike coefficients $\mathbf{z}_{ref}$ from this map, using a rotated and translated coordinate systems. The circle shows an example of modified pupil area over which we computed $\mathbf{z}_{ref}$, for a shift [$t_x = 0.15$ mm, $t_y = -0.15$ mm] and a rotation $\theta = 2$ degree. The pupil diameter that we use to compute $\mathbf{z}_{ref}$ is 5.25 mm, as in Table 1.

Fig. 5. Estimated positions of the centre of the pupil (left) and the angle of rotation (right) obtained with the first plate, after modifying the coordinate system of $\mathbf{z}_{ref}$ as we illustrated in Figure 4.

For four positions of the pupil, we also numerically introduced rotations of the PSI measurements. We show in Figure 5 (right graph) the very good linearity in the estimated angle, with a slope of 1.00. The different curves correspond to an additional (fixed) translation of ± 0.15 mm in the vertical and horizontal directions. A small offset between the curves ($< 0.1^\circ$) show that the estimation of the angle θ is partially coupled to the position of the pupil centre.

4. Conclusions and acknowledgement

We have presented a method to cancel the alignment artefacts when comparing two accurate measurements of the same optical system. For this application, we found an accuracy in the order of 10 microns (translation), 0.1 degree (rotation), and 1 % (scale), provided that the pupil over which we compare the wavefronts is entirely contained by the original pupils. There is no need for additionnal prior knowledge of the misalignment parameters. The relative insignificance in the choice of the values ($t_{x,0}$, $t_{y,0}$, θ_0, γ_0) is a consequence of the large dimension of the data space (63 Zernike coefficients, compared to 4 estimated alignment parameters). Further investigations would be required to quantify the robustness of this method to intrinsic difference between the wavefront measurements. This method has a potential value for the design of optical systems, as it can decrease the mismatch between simulations and measurements of optical performance.

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