EXPERIMENTAL DETECTION OF OPTICAL VORTICES USING A SHACK-HARTMANN WAVEFRONT SENSOR

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1. Background

Optical vortices (also know as branch points or phase singularities) occur when the phase of an optical field becomes undefined in a region of zero amplitude. The phase of the wavefront is discontinuous about the vortex and with a $2m\pi$ radian jump across the discontinuity, where m is an integer number corresponding to the charge of the vortex. This leads to a spiral phase change going from 0 to $2m\pi$ around the undefined phase at the centre, [Fig. 1]. Vortices occur in pairs which consist of both a positive and a negative vortex with the sign determined by the direction of the rotation of the spiral phase pattern around the centre of the vortex. The wavefront dislocation which joins the two oppositely signed vortices is along the line where the $2m\pi$ discontinuity is present and is sometimes referred to as a branch cut.

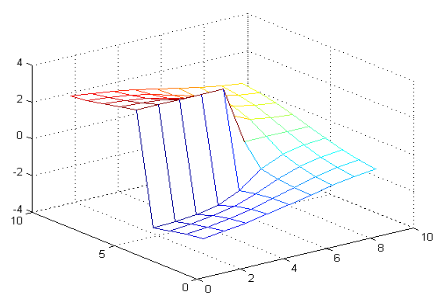

FIGURE 1. A 2π radian vortex. This shows the spiral phase around the centre of the vortex as well as the wavefront dislocation created by the vortex.

There is much interest in trying to correct for the distortions induced the atmosphere on a propagating laser beam of which optical vortices are a large part, with one the main techniques for accomplishing this being Adaptive Optics (AO). AO systems look to measure the wavefront and correct for it's aberrations by using a controllable optical element and generally consist of a wavefront sensor, a camera, a deformable element and a control system along with static optical elements. Optical vortices can cause a number of difficulties for AO systems, in both the hardware and the control system, which try to correct for them.

The operation of the Shack-Hartmann wavefront sensor is shown below [Fig.2]. Initially a plane wavefront is passed through the lenslet array and falls onto the detector. The lenlets split the beam up into the same number of beamlets and these will all form an in-focus spot on the detector. The position of these spots are noted and called the reference positions. Then when an aberrated wavefront is passed through the lenslet array another set of spot positions are noted. The difference in these spot locations compared to the reference spot positions gives us the phase gradients (or slopes) of the aberrated wavefront we are measuring.

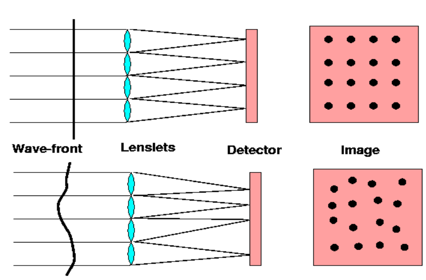

FIGURE 2. Operation of a Shack Hartmann wavefront sensor.

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2. Experimental Setup

An experimental setup was built in the laboratory to conduct the practical tests. A schematic of the setup is shown in [Fig. 3]. The system is configured with two parallel beams which, after coming through a spatial filter from the laser, are split by a beamsplitter (BS1). One of these beams is a reference beam which is sent through a 4f system to the second set of beamsplitters (BS3,BS4), through the optical trombone and back into the wavefront sensing arm. When Shack-Hartmann wavefront measurements are being carried out this beam is blocked off somewhere in the path before it rejoins the Spatial Light Modulator (SLM) beam. The Spatial Light Modulator is a ferroelectric liquid crystal, binary phase SLM from Boulder Nonlinear Systems with 512 x 512 pixels and an active area of 7.7mm x 7.7mm.

The second beam going from BS1 goes through another beamsplitter (BS2) and is directed onto the SLM. After being reflected from the SLM the beam comes back through BS2 and into a 4f system, which has an iris placed at the focus to select only one diffraction order of the SLM. It then travels to BS4, where it rejoins the reference beam, and through the optical trombone. It then proceeds back through into the wavefront sensing arm. In the wavefront sensor itself a lens (L8) is used to expand the beam slightly to ensure it passes through the active area of the Shack-Hartmann lenslet array. Due to the relatively short focal length of the lenslet array a relay lens (L9) is needed to focus the beamlets on to the camera.

Another arm is used for interferometry, this arm starts with a flip mirror (FM1). When interferometric images are needed this flip mirror is dropped into the path of the beam before the wavefront sensing arm. It redirects the beam into lenses L10 and L11 which demagnify the beam size onto the CCD used for interferometry. When measurements involving the Shack-Hartmann sensor are needed the mirror FM1 is flipped out of the beam path and the beam continues into the Shack-Hartmann wavefront sensing arm.

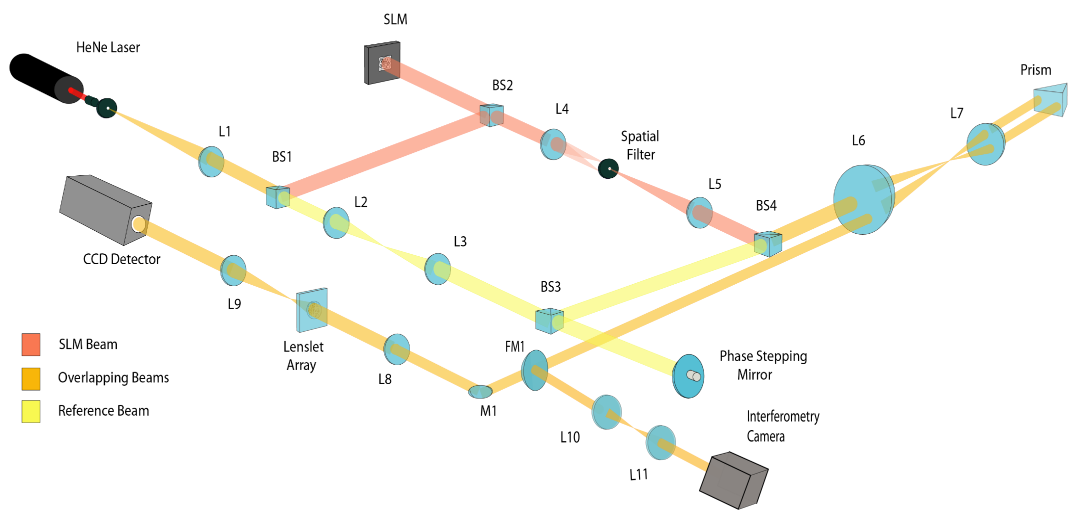

Figure 3. Lab setup.

Part of this laboratory setup had been used previously within the group in a different project as a tabletop atmospheric turbulence generator. The laboratory turbulence generator was slightly modified and the measurement arms for both the Shack-Hartmann wavefront sensor and for the interferometry were added. For further information on the other project mentioned, Adaptive Optics correction in strong turbulence, please click on the link here.

To detect optical vortices in atmospheric like conditions we use the SLM to generate amplitude and phase variations in a field typical of those encountered after the propagation of a laser beam through the atmosphere. These turbulence maps were created by numerical simulations which passed a laser beam through 15 phase screens randomly generated using Kolmogorov statistics. The light source was propagated between phase screens by multiplying the Fresnel transfer function by the Fourier transform of the optical field. To ensure that an optical vortex is present in the final turbulence map the initial phase screen is seeded with an optical vortex of known sign. More research on the dynamics of optical vortices being conducted within the group can be seen here.

3. Vortex Detection

The slopes (or phase gradients) calculated from a SHWFS are related to the optical wavefront by the following

(1) $$s = A\phi$$

where A is a matrix which relates the phase gradients, s, to the wavefront phase, ϕ.

The least mean square error reconstruction, ϕ_{lmse}, can be written in the following form

(2) $$\phi_{\text{lmse}} = ((A^{\dagger}A)^{-1}A^{\dagger})s$$

This however does not take account of all the phase in the reconstruction process, some of the phase is ignored and treated as noise. This is called the hidden phase and this is the part of the phase which is discontinuous when vortices are present. It is defined as

(3) $$\nabla\phi \;=\; \nabla\phi_{\text{lmse}} \;+\; \nabla\phi_{\text{hid}}$$

where $\nabla\phi$ is the gradient of the phase of the field, $\nabla\phi_{lmse}$ is the gradient of the least-squares phase and $\nabla\phi_{hid}$ is the gradient of the hidden phase.

The hidden phase is the part of the phase which is the curl of the vector potential (the least squares reconstruction only looks at the scalar potential) and thus can be defined as

(4) $$\nabla\phi_{\text{hid}} \;=\; \nabla \;\times\; H(r)$$

where the H(r) above is the Hertz potential, which we will take advantage of when we detect vortices in a distorted phase function.

The method we use to detect optical vortices, called the branch point potential method, determines the Hertz potential of the field. It is done by first rotating the phase gradients calculated from the wavefront sensor by 90 degrees.

(5) $$(R_{\pi/2}) * (s) = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} * (s) = \begin{pmatrix} s_y \\ -s_x \end{pmatrix}$$

where $(R_{\pi/2})$ is a 90 degree rotation, s are the slope values measured by the wavefront senor with s_x and s_y the slopes x-values and y-values respectively.

Then the pseudo-inverse of the geometry matrix is calculated:

(6) $$A' = (A^{\dagger}A + m^2)^{-1}A^{\dagger}$$

where $A^{\dagger}$ is the adjoint of the geometry matrix and the m^2 value is inserted to prevent the matrix from becoming singular. The potential function, V is then calculated by multiplying the rotated slopes by the pseudo-inverse of the geometry matrix.

(7) $$V = (A')(R_{\pi/2}\ s)$$

The potential function, V, is then reformed into a square array of the same size as the Shack-Hartmann lenslet array and the position and sign of the vortices is found by locating the peaks or valleys in the potential function. The positive vortices are designated by the peaks in the potential plot and the negative vortices are designated by the valleys. This potential function can be used to locate the optical vortices because it is the Hertz potential Eq. 4 and therefore it is able to see the hidden phase of the field.

We have found that this method can be improved upon by using it along with the slope discrepancy technique. This is when the slope field is given by subtracting the SH slopes from the slopes obtained from a least squares reconstruction of phase

(8) $$\nabla\phi_{\text{slpdis}} \;=\; s \;-\; \nabla\phi_{\text{lmse}}$$

where $\nabla\phi_{\text{slpdis}}$ are the phase gradients of the slope discrepancy, $\nabla\phi_{lmse}$ are the phase gradients of the least-squares phase and s are the phase gradients calculated from the wavefront sensor.

The slope discrepancy phase gradients are then used as the inputs into the branch point potential detection method and when these are used instead of the Shack-Hartmann slopes we get improved vortex detection. This happens because tilts and other large aberrations that are present in the wavefront have a strong contribution to the slope data and can mask the relatively small circulation of the slopes when a vortex is present By removing the continuous part of the phase from the slope data the vortex circulation becomes more apparent. Since these large aberrations are essentially noise to our vortex detector we get increased levels of detection.