Wavefront sensing and adaptive optics in strong turbulence

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ABSTRACT
When light propagates through the atmosphere the fluctuating refractive index caused by temperature gradients, humidity fluctuations and the wind mixing of air cause the phase of the optical field to be corrupted. In strong turbulence, over horizontal paths or at large zenith angles, the phase aberration is converted to intensity variation (scintillation) as interference within the beam and diffraction effects produce the peaks and zeros of a speckle-like pattern. At the zeros of intensity the phase becomes indeterminate as both the real and imaginary parts of the field go to zero. The wavefront is no longer continuous but contains dislocations along lines connecting phase singularities of opposite rotation.

Conventional adaptive optics techniques of wavefront sensing and wavefront reconstruction do not account for discontinuous phase functions and hence can only conjugate an averaged, continuous wavefront. We are developing an adaptive optics system that can cope with dislocations in the phase function for potential use in a line-of-sight optical communications link.

Using a ferroelectric liquid crystal spatial light modulator (FLC SLM) to generate dynamic atmospheric phase screens in the laboratory, we simulate strong scintillation conditions where high densities of phase singularities exist in order to compare wavefront sensors for tolerance to scintillation and accuracy of wavefront recovery.

Keywords: Adaptive Optics, Strong Turbulence, Scintillation

1. INTRODUCTION
When light propagates through the atmosphere fluctuations in refractive index caused by temperature gradients and wind mixing of air induce phase and amplitude fluctuations in the propagating field. In weak turbulence amplitude fluctuations can be ignored and it is possible to remove the effects of the atmosphere by conjugation of the phase perturbation using a deformable mirror. This technique has proved very successful in ground-based astronomy, allowing images from telescopes on Earth to exceed those of the Hubble space telescope in terms of angular resolution.

In recent years, there has been increasing interest in applying adaptive optics in strong turbulence conditions, particularly for applications in line-of-sight free-space optical communication. In this regime conventional adaptive optics techniques of wavefront sensing and phase conjugation break down as amplitude fluctuations can no longer be ignored. As the field propagates through strong turbulence multiple scattering occurs randomising the phase to produce speckle-like patterns of intensity. At the zeros of intensity the phase becomes undefined as the real and imaginary parts of the field are zero. The phase is no longer continuous but contains phase dislocations along lines joining phase singularities of opposite rotation.

In this paper we discuss propagation of light through strong turbulence and describe the laboratory turbulence generator we have built as a test bed for development of an adaptive optics system. Our initial aim is to compare wavefront sensors for robustness to scintillation and the ability to detect phase singularities. In section 2 we discuss the effects of propagation through strong turbulence and show through numerical simulation the onset of phase dislocations. In section 3 we describe the laboratory turbulence generator and in section 4 we discuss methods of wavefront sensing and phase reconstruction in the presence of phase singularities.
2. PROPAGATION THROUGH STRONG TURBULENCE

In weak turbulence Rytov theory of smooth perturbations gives an analytic solution to propagation through the atmosphere. After propagation through a thin atmospheric layer the field has an additional phase term \( \exp\{i\psi_{\text{layer}}(r)\} \), which when allowed to propagate to a receiver plane introduces a log-amplitude fluctuation \( \chi \) and a phase fluctuation \( \phi \). The log-amplitude and phase fluctuations are normally distributed and statistically independent of each other.\(^5\) Rytov theory predicts that the fluctuations in log-amplitude, \( \sigma^2_{\chi,\text{Rytov}} \), should increase as the propagation distance or turbulence strength increases. However, it is found that log-amplitude fluctuations begin to saturate for values of \( \sigma^2_{\chi,\text{Rytov}} > 0.3 \). This marks the beginning of departure from the weak fluctuation approximation. In this regime Rytov theory is no longer valid and there is no analytic solution to propagation through the atmosphere. In order to perform analysis in strong atmospheric turbulence conditions simulation must be done, either numerically or in the laboratory.

2.1. Numerical Simulation

In this section we summarize and combine methods from several papers\(^{6-8}\) on simulating propagation through optical turbulence in order to demonstrate the effects of propagation through strong turbulence.

The atmospheric fluctuations in phase can be represented by a series of thin phase screens of thickness \( dz \). Where the term thin implies that no diffraction occurs within the phase screen. The phase screens are produced to follow the Kolmogorov power spectrum of fluctuations,\(^{9-11}\)

\[
\hat{\Phi}(\kappa) = 0.033 C_n^2 \kappa^{-11/3}
\]

where \( \kappa \) is the spatial frequency and \( C_n^2 \) is the refractive index constant and indicates the strength of the turbulence. This is valid over the inertial range, \( \frac{1}{L_0} < \kappa < \frac{1}{\kappa_0} \), where \( L_0 \) is the outer scale and \( l_0 \) the inner scale of the turbulent cells. To simulate more accurately the effects of the inner scale in strong turbulence the Von Karman spectrum is used,

\[
\hat{\Phi}(\kappa) = 0.033 C_n^2 \frac{\exp(-\kappa^2/\kappa_m^2)}{(\kappa^2 + \kappa_m^2)^{11/6}},
\]

where \( \kappa_m = 5.92/l_0 \) and \( \kappa_0 = 2\pi/L_0 \).

For horizontal propagation paths values of \( C_n^2 \sim 10^{-15} m^{-2/3} \) to \( 10^{-15} m^{-2/3} \) can be expected.\(^{12}\) In order to ensure that the propagation distance between phase screens remains in the weak turbulence regime, the interscreen distance \( z \) is chosen so that the variance in log-amplitude, \( \sigma^2_{\chi}(z) \) is less than 0.3. If we assume a constant \( C_n^2 \) profile over the total propagation distance the log-amplitude fluctuation at distance \( L \) is\(^{13}\)

\[
\sigma^2_{\chi}(L) = 0.307 k^{7/6} l^{11/6} C_n^2.
\]

The interscreen propagation distance \( z = L/n_s \) (where \( n_s \) is the number of screens) remains in the weak turbulence regime for \( \sigma^2_{\chi}(z) < 0.3 \). By taking the log-amplitude fluctuations for the whole path and letting \( \sigma^2_{\chi}(z) = 0.1 \) we find the number of screens to use.

\[
\sigma^2_{\chi}(z) = \frac{\sigma^2_{\chi}(L)}{n_s^{11/6}} = 0.1
\]

\[
n_s = \left[10 \sigma^2_{\chi}(L)\right]^{6/11}.
\]

Propagation between phase screens is done using the Fresnel propagator.\(^{14}\) Fresnel diffraction is performed by conjugating the perturbed field with the Fresnel kernel. In the Fourier domain this is equivalent to multiplication of the Fourier transform of the field with the Fresnel transfer function,

\[
H(\kappa_x, \kappa_y) = \exp\{iz\lambda\pi(\kappa_x^2 + \kappa_y^2)\}.
\]
Figure 1. Numerical propagation simulation. The wrapped phase values (−π, π) of a plane wave with diameter 20 cm after propagation through (a) 1125m, (b) 1875m, (c) 2625m, (d) 3000m, of atmospheric turbulence with strength $C_n^2 = 10^{-14} \, m^{-2/3}$. The 3 km path is represented by 4 phase screens with an interscreen propagation distance of 750m. Phase dislocations are evident by the broken contours of the wrapped phase and become apparent from 1875m onwards.

To ensure the argument in eq.6 is sampled at greater than the Nyquist rate at the maximum spatial frequencies, the grid of $N \times N$ pixels with pixel width $\Delta_x$ is chosen so that

$$N > \frac{2z\lambda}{\Delta_x^2}. \quad (7)$$

Figure 1 shows the wrapped phase values of a plane wave with $\lambda = 633\,nm$ over a propagation path of 3 km through turbulence of strength $C_n^2 = 10^{-14} \, m^{-2/3}$. The 3 km path is represented by 4 Kolmogorov phase screens separated by 750m. The propagation was done on a grid of 256 x 256 with pixel width of 2mm. Note that the propagating field is masked with a super-gaussian apodizing mask (with radius of the beam diameter) after propagation between the phase screens. This is done to remove the poor wavefront sampling at the edges of the beam. The phase dislocations, which can be seen by the broken contours in the wrapped phase, begin to appear after 1 to 2 km of propagation.

3. SIMULATION OF ATMOSPHERIC TURBULENCE IN THE LABORATORY

To generate optical turbulence in the laboratory we apply an atmospheric phase screen to a ferroelectric liquid crystal spatial light modulator (FLCSLM). The advantage of this device is that it is reconfigurable at a rate of 1kHz, which is the speed needed to simulate real-time evolution of the atmosphere. As the device is only capable
Figure 2. Turbulence generator and phase-stepping interferometer. A plane tilt is combined with a phase screen and binarized to form a diffraction grating on the FLCSLM. In the back focal plane of the first lens the first positive diffraction order is spatially filtered and Fourier transformed by the second lens to produce the analogue phase screen. By allowing the phase screen to propagate beyond the back focal plane of L2 we are able to simulate atmospheric turbulence induced scintillation. The reference beam and phase-stepping mirror allow us to retrieve the phase from an interference pattern.

of binary phase modulation we use a technique from computer generated holography, as described by Neil et al.,\textsuperscript{15,16} in order to produce an analogue phase screen from the binary phase modulation of the FLCSLM.

The method to generate the diffraction grating is described in brief here. After generating a phase screen we then add a linear phase tilt. The combined phase modulo $2\pi$ is binarized and applied to the FLCSLM. The binarized phase $\phi_{\text{bin}}$ is defined by

$$\phi_{\text{bin}} = \begin{cases} 0 & \text{if } \operatorname{mod} \left( \frac{\phi}{\pi} \right) < \pi \\ 1 & \text{if } \operatorname{mod} \left( \frac{\phi}{\pi} \right) > \pi \end{cases}$$

The linear tilt creates a carrier frequency for the phase screen, which when spatially filtered in the back focal plane of lens 1 (see fig. 2) and Fourier transformed by the second lens produces the analogue phase screen with a tilt. After generating the phase screen it is a simple matter of moving the CCD camera beyond the back focal plane of lens L2 to allow the scintillation to develop. We use a phase-stepping Mach-Zender interferometer to measure the phase produced. There is an rms error of approximately $\lambda/4$ in the reconstructed wavefront when compared with the original phase screen, predominantly due to variations in the displacement of the phase-stepping mirror.

The phase is retrieved modulo $2\pi$ and must be unwrapped to recover the wavefront. Reconstructing the wavefront from the discontinuous phase requires locating the branch points and isolating them from the phase unwrapping process (see section 4 for a more full description of the significance of branch points). One method is to apply the Goldstein algorithm as discussed by Roggemann et al.,\textsuperscript{8,17} and by Ghiglia and Pritt.\textsuperscript{18} The branch points are located by doing contour sums of the wrapped modulo $2\pi$ phase differences. For a loop containing a branch point the sum will give $\pm 2\pi$. The Goldstein method involves locating all the branch points and connecting singularities of opposite rotation to form a branch cut. The phase is then unwrapped along paths that do not cross branch cuts.

In figure 3 it is possible to see the phase induced scintillation and the interference fringes produced when the reference beam is allowed to interfere with the aberrated beam. Note the appearance of fringes at the zeros of intensity due to the $\pi$ phase change across the singularity.
Figure 3. The figure on the left is the phase induced scintillation created by the FL-CSLM after propagation of a phase screen with $D/r_0 = 40$. The equivalent propagation distance simulated for an $r_0 = 2.5\text{cm}$ is $1\text{km}$. The figure on the right shows the interference fringes when the scintillated beam is combined with the reference beam. A region of zero intensity is circled in white in both figures to highlight the presence of a phase singularity. An additional fringe indicates the $\pi$ phase change as one crosses a phase singularity.

4. WAVEFRONT SENSING IN STRONG TURBULENCE

Conventional adaptive optics systems rely on the absence of scintillation to determine the wavefront and reconstruct a continuous phase deformation. In strong turbulence the spatial intensity variation makes it difficult to make measurements of the wavefront and the presence of phase singularities at zeros of intensity introduces an additional error in the phase measurement, as the phase is no longer continuous. Using the laboratory turbulence generator we plan to investigate the ability of wavefront sensors to correct scintillated and discontinuous wavefronts.

Recently, Barchers has reviewed the performance of several wavefront sensors in strong turbulence\textsuperscript{19–21} and has suggested that a point diffraction interferometer is perhaps the best suited to strong turbulence conditions.

Ideally we would like to use a simple and robust wavefront sensor such as the Shack-Hartmann wavefront sensor (SHWFS), which consists of a lenslet array and CCD camera positioned at the back focal plane of the lenslet array. The average gradient of the phase across each lenslet is retrieved by measuring the displacements of the focal spots from a reference position. The SHWFS is widely used in adaptive optics systems because of its computational and physical simplicity. However, when the lenslet dimension is on the order of the atmospheric coherence length, $r_0$ the ability of the SHWFS to reliably recover the wavefront is greatly reduced due to scintillation and the presence of branch points.\textsuperscript{19} Despite this, the benefit of low-order adaptive optics correction to a beam propagating along horizontal paths through strong turbulence has been demonstrated in laboratory simulation by Tyson et al.\textsuperscript{22, 23} and if the branch points in the phase function can be identified and removed the experimental results of Primmerman et al. and Levine et al.\textsuperscript{24, 25} suggest that low-order adaptive optics correction with a SHWFS is possible in moderate to strong turbulence over paths of several kilometers.

Fried has investigated the problem of branch points in adaptive optics\textsuperscript{4, 26, 27} and has derived an analytic solution for the rotational part of the phase, which is hidden from the typical least squares reconstructor used to reconstruct the wavefront from the measured phase gradients of a SHWFS.

The gradient of the phase can be written as

$$\nabla \phi = \nabla \phi_{\text{lmse}} + \nabla \phi_{\text{hid}},$$

where $\nabla \phi_{\text{lmse}}$ is the curl-free phase gradient that is found by the least mean squares reconstructor and $\nabla \phi_{\text{hid}}$ is the hidden phase gradient and is equal to the curl of a potential,
∇φ_{hid} = ∇ \times \mathbf{H}(r). \quad (9)

As mentioned previously in section 3, branch points can be located by doing contour sums of the wrapped phase differences around a closed loop. If the loop encloses a branch point there will be an extra ±2π. The potential only has a z component \(\mathbf{H}(r) = [0, 0, h(r)]\). Taking the curl of \(\mathbf{H}(r)\) gives

\[
\nabla \times \nabla \times \mathbf{H}(r) = -\nabla^2 h(r) = \pm 2\pi \delta(r - r_{bp}), \quad (10)
\]

where \(r_{bp}\) is the position of the branch point.

Fried has derived an analytic solution to retrieve the rotational part of the phase.\(^{26}\) We simply state the solution here for a single branch point at location \((x_{bp}, y_{bp})\). The hidden phase contribution to the total phase is

\[
\phi_{hid} = \text{Im}\{\pm \log[(x - x_{bp}) + i(y - y_{bp})]\}. \quad (11)
\]

5. SUMMARY

In strong turbulence first order perturbation approximations for wave propagation through the atmosphere are no longer valid. In this regime numerical simulation or laboratory simulation must be used to determine the phase and intensity statistics. In strong turbulence nulls in intensity can appear causing the phase at that point to become indeterminate. These phase singularities are not detected by the usual least mean squares estimator used to reconstruct the wavefront from phase gradients. Using a branch point sensitive reconstructor it is possible to reconstruct the discontinuous phase. We have described the laboratory simulation we are using in order to compare wavefront sensors for robustness to scintillation and discussed briefly the topic of wavefront sensing and wavefront reconstruction in strong turbulence.

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REFERENCES


