MODELLING OF NONSTATIONARY DYNAMIC OCULAR ABERRATIONS

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The human eye is an optical system with its own characteristic aberrations. It has been well established that the aberrations of the eye are dynamic - i.e. that they vary over time. The goal of this work is to develop methodologies for computer modelling and simulation of these dynamic aberrations. Dynamic aberrations can be measured experimentally using a wavefront sensor. Analysis shows that the time-series for the various aberration modes are nonstationary. This makes the goals of analysing and modelling the process more difficult. An Autoregressive Integrated Moving Average (ARIMA) modelling approach is described, and results are presented.

Keywords: Ocular aberrations; ARIMA Models; Nonstationary Processes

1. Introduction

As an optical system, the human eye has its own characteristic aberrations, which occur mainly due to the surface of the cornea and the lens. These take the form of both chromatic and monochromatic aberration types. In this research we are concerned with the latter. The dynamic nature of ocular aberrations has been widely investigated [1-4], for example by Hofer et al. [1], who presented results illustrating these dynamic effects, and suggested some of the possible causes. In particular, it has been shown that the tear film has significant dynamic effects on the overall aberration of the eye [5, 6]. Other possible factors include, eye tremors, microfluctuations in accommodation, and retinal pulsation [3].

A useful model of static aberrations in the human eye was developed by Thibos et al. [7, 8]. This model established that in general, higher-order aberrations tend to be normally distributed around zero for a large population of eyes. This leads to a reasonable assumption that the aberration function for an individual eye can be modelled as a multivariate Gaussian
random variable with known mean, variance, and covariance. The paper suggested the use of a model to create a database of “virtual eyes”. The approach was based on a static model however, i.e. the dynamic nature of the eye’s aberrations were not taken into account. Our work aims to develop a modelling approach that also accounts for dynamic effects.

2. Measurements

To analyse the aberrations of the human eye, the concept of the wavefront aberration is often used. A wavefront is the locus of electric field points of equal instantaneous phase. The wavefront aberration of the eye can be estimated from measurements performed in the laboratory, for example using a Shack-Hartmann sensor [9]. In the procedure, the test subject firstly has his/her head secured in place using a bite-bar. Tropicamide is normally used to paralyse the eye’s accommodation. The subject’s retina is illuminated using a near infra-red laser source, and the back-scattered light is then sensed using a Shack-Hartmann wavefront sensor. The length of dataset taken varies, but short datasets can be easier to work with as it is possible for the subject to avoid blinking for the duration of the test. Blinking introduces discontinuities and complicates the results, as can be seen in Figure 1.(b). The Shack-Hartmann spot positions are read out to a PC through a CCD camera.

In order to derive meaningful interpretations from the data, the local wavefront slopes are estimated from the Shack-Hartmann spot measurements. This in turn enables the calculation of the corresponding estimated wavefront. It should be noted that the presence of speckle and electronic noise has an adverse effect on these estimations. A method of performing these calculations is described in [10]. A centroiding algorithm is used to define the exact position of each spot on the CCD. Each spot can be defined by horizontal and vertical shifts of its focal point. The local wavefront slopes can then be estimated, and used to determine the coefficients in a Zernike polynomial expansion, which can be described by Eq. (1):

$$\phi(r, \theta) = \sum_{k=1}^{M} a_k Z_k(r, \theta)$$

where $\phi$ denotes phase, $M$ is the number of terms used in the expansion, and $a_k$ are the Zernike coefficients.
3. Nonstationary Processes

It is important to consider stationarity when performing statistical analysis on a process. A process $X(t)$ is said to be strictly stationary if for any delay $\tau$, the joint probability density of $\{X(t_1), X(t_2), \ldots, X(t_n)\}$ is identical to the joint probability density of $\{X(t_1 + \tau), X(t_2 + \tau), \ldots, X(t_n + \tau)\}$. This implies that the statistical properties of the process are independent of time. It was clear from analysis of the eye aberration data that this was not the case. Therefore it can be stated that the dynamic aberrations of the eye fail to meet the criteria for either strict stationarity or wide-sense stationarity (which places restrictions just on the first and second order moments). This result means that the dynamic ocular aberrations of the eye should be classified as a nonstationary process. This makes the analysis and modelling significantly more difficult [11].

4. Modelling

We proceed to develop a statistical model of the dynamic aberrations of the eye, based on observed data. It should be noted that ideally one should take a large number of observations from many different subject eyes as the basis for modelling. In this paper just one subject is used, to illustrate the procedure. The resultant model is specific to that subject.

4.1. Autoregressive Integrated Moving Average (ARIMA) Models

The ARIMA approach is a generalised case of the standard ARMA model as described by Box and Jenkins, which extends it to include certain classes of nonstationary processes [12]. In the stationary ARMA case, the aim is
to model a discrete process \( y \) as the output of a causal linear filter driven by white noise \( \nu \) as follows [13]:

\[
y(n) = - \sum_{k=1}^{p} a_k y(n - k) + \sum_{k=1}^{q} b_k \nu(n - k)
\]

(2)

The \( p \) and \( q \) limits determine the order of the model. \( a_k \) and \( b_k \) are the model coefficients and can be estimated by solving linear equations constructed from autocorrelation calculations of the process. This is known as the Yule-Walker (or autocorrelation) method.

For the ARIMA case, a level of differencing \( d \) is applied to the data to render it stationary. After differencing, the autocorrelation method can again be used to extract ARMA parameters, giving the completed model:

\[
z(n) = y(n) - y(n - d) = - \sum_{k=1}^{p} a_k y(n - k) + \sum_{k=1}^{q} b_k \nu(n - k)
\]

(3)

Often the choice of the model order and differencing is not straightforward. By looking at the sample autocorrelation (SAC) and sample partial autocorrelation (SPAC) functions, one can gain insights about the nature of the process and make a more informed choice of the ARIMA parameters. The partial autocorrelation of a time series at lag \( k \) can be thought of as the autocorrelation at lag \( k \) with the effect of intervening observations removed. Figure 2.(a) shows a measured dataset (in this case Zernike total RMS error with blink discontinuities removed), along with its SAC and SPAC as calculated using R. The slow decay of the SAC is a characteristic often observed in nonstationary processes.

### 4.2. Simulation of Models

Simulation is carried out by driving the completed model with a Gaussian white noise process. This converts the input sequence to a process with the same characteristics as the true process used to obtain the model. In the ARIMA case, after simulation the nonstationary data can be recovered from the ARMA simulated data by taking partial sums [14]. Figure 2 shows results from a simulation performed using R. In Figure 2.(a), an ARIMA \((3,1,3)\) model was obtained from a set of measured data, then this model was simulated as described above. The resulting output, shown in Figure 2.(b), has similar characteristics to the original process, as can be seen both from the time series themselves, as well as the similar SAC and SPAC plots. It should be remembered that the simulated model output is not intended
to be identical to the original data, but merely to be statistically similar, i.e. their mean, variance and autocorrelation functions should be comparable.

5. Discussion

A procedure to model dynamic ocular aberrations and to simulate aberration data has been outlined. It is hoped that this technique could be useful in areas such as testing of optical designs, simulation of retinal image quality or visual performance, or for control design or prediction in adaptive optics systems.

It should be noted that although it is assumed for the construction of the model that the various Zernike modes are independent of each other, there is some evidence that weak correlations exist between certain modes [7]. It is planned to investigate these correlations further and to include their effect in the model if required. It has also been shown that there exists a weak correlation between the cardiopulmonary system and eye aberrations [4]. Since aspects of this can be measured experimentally, it may be possible to include its effects in the model as an additional regressor. Further investigation is required.

It is planned to improve on this modelling technique in the future. In
particular, it is hoped to collect a large database of aberration measurements to produce a more comprehensive model that could take into account variations over sample population, as well as over time.

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References